# When private education betters long-run public investment in human capital

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## 1. Introduction

Public education is a good asset to reduce poverty and inequalities in the society. According to classical political theory, inequalities as they lower the position of the median voter in the society, should favor public investment in education. However, many poor countries, like Peru, seem reluctant to improve their public education. Many actors of the civil society in Peru invoke Marxist theories of class struggle to explain the lack of public investment. A powerful upper class could keep the political power by preventing the poorest citizens from acquiring human capital and gaining economic independence. Nevertheless, the situation in Peru, which is a democracy, may be much more complicated. This paper aims to give an explanation of endemic low investments in public education consistant with microeconomic evidences and the political theory. It also shows that introducing private education could be a way to solve this problem.

### *1.1. Empirical findings on education demand in Peru*

There is a large consensus in the Peruvian civil society on the necessity to increase public expenditure in education to promote economic growth and decrease poverty. Despite of the popularity of this isssue in the electoral contests the public contribution to education is still very weak in Peru. Therefore, the quality of public education remains low, as withnessed by the statistics and the public opinion. In 2007 according to the EVEP survey (see below), 42.7% of peruvian considered the quality of public education to be insufficient, 46.2% was not satisfied from their education, 29.5% have repeted at least once and 72.0% would support the increase of hours of schooling.

| Year | Before 1970 | 1970 | 1975 | 1980 | 1985 | 1990 | 1995 | 2000 | 2005 |
|---|---|---|---|---|---|---|---|---|---|
| Pub. expenditure | – | 3.2 | 3.3 | 2.9 | 3.2 | 3.2 | 3.2 | 2.9 | 2.6 |
| Private ed. Peru | 5.0 | 7.9 | 9.2 | 4.8 | 9.2 | 11.2 | 13.6 | 16.9 | – |
| Private ed. Lima | 6.9 | 10.5 | 13.3 | 7.2 | 16.4 | 16.1 | 23.3 | 27.3 | – |

Table 1: Public education expenditure$^{(*)}$ and private school attendance$^{(+)}$ in Peru.
$^{(*)}$ (as % of GDP) five year average except for 1970 and 1975, World Bank.
$^{(+)}$ Share of interviewed people by year of their 12th birthday, EVEP survey 2007

Whereas than trying to improve the public system, Fujimori's government in the 90's had on the contrary widely supported the developement of the private education supply. Private education has indeed kept growing, especially in the main cities and Lima. However, private education seems not to have crowded out public expenditure for education which have remained very stable, around 3 % of GDP since the 70's (see table 1).

According to the data, this strategy has effectively fasten the human capital accumulation although many actors of the civil society complain that it has weakened the political claims of population for better public education. As the quality of public education in Peru is highly undermined by the lack of public expenditure, the difficulties of public education and the support to the private supply is clearly a political problem. Despite of several substantial change in governments since the 50's[1], the country has never been able to improve the quality of its public schools. An explanation could be that there has been until today no political consensus supporting higher education expenditure because these would imply with higher taxes.

To check this theory, this paper introduces the results of an original survey on education demand in Peru, EVEP 2006-2007[2], based on the last household survey made by the World Bank in Peru in 1994, ENNIV. The aim of this survey was to characterize the demand for education in Peru and to calibrate a human capital growth model. The survey was conducted on 1719 individuals in two waves, in the provinces of Lima and Puno in 2006 and in the provinces of Cuzco and Huancayo in 2007 by the same enquirers, using the same methodology and questionnary. Since asked questions concerned the education and personal characteristics of the interviewed persons such as composition of family or conditions of childhood, the delay between the two waves does not cause problems of data comparability. The only variable which could be corrected is declared income. However, inflation was controled in 2006-2007 in Peru and moreover the given answers on income are rather approximative.

| Area | Income per household (current soles) | Income per worker (current soles) | Schooling (years) | Sex (female) | Age (years) | Quecha spoken | Households (number) |
|---|---|---|---|---|---|---|---|
| Rural | 448 | 267 | 8.4 | 52.7% | 33.6 | 69.2% | 451 |
| Urban | 1139 | 586 | 11 | 51.5% | 33.4 | 21.0% | 1268 |

Table 2: Main statistics of the EVEP survey

The survey exhibits three types of indicators aiming to characterize the demand for education. First the participation can be described by drops out before the end

[1]Peru has successively known a socialist revolution brought by the militaries in the 70's (Velasco), conservative (Belaunde) then radical (Garcia) democratic elected governments in the 80's, a pro-market but autoritarist government in the 90's (Fujimori) and a center left one in the 2000's (Toledo).

[2]The Encuesta sobre la Vision de la Educacion en el Peru has been conducted by the author and several professional enquirers in the year 2006 and 2007 thanks to the collaboration of the university Antonio Ruiz de Montoya of Lima.

of compulsory school. Second, two questions were asked to measure the priority of the expenditure in education. In the first one, interviewed people had to decide how the government should split the revenues of an additional tax of 100 soles monthly per household. In the second one, people were asked how they would spent an additional income of 100 soles monthly they earned by chance. Third, people should declare the amount of money they really spent for their children's education and the amount they would spend if they could benefit from a long-term loan to pay for their children's education. The following table shows regressions of those answers on income, years of schooling and area of residence. IV regressions have been used in order to get rid of endogeneity problems with variables such as savings or insufficient alimentation.

These findings seems to indicate that whatever the indicator and the specifications, the propensity of parents to invest in their elder's education increases with their level of education. Moreover, there seems to be a threshold level in the preference for education at the university level. Having attended university increases indeed every indicators, whatever college studies lasted.

The assumption of an increasing propensity to invest in education with education attainment can be related to the one of increasing propensity to invest with income This very old hypothesis, Fischer (1930), Kaldor (1955) has been recently verified on US data by Lawrence (1991). Many contemporary contributions make this assumption, see Becker and Mulligan (1997), Samwick (1998) or Atkenson (1997).

According to EVEP data, more educated people are more likely to pay for their children's education, whether through a tax or a private contribution. On the one hand, the poverty rate is too low to allow the emergence of a political majority supporting high tax for high expenditure in public education. On the other hand, the poor quality of public education prevent the country to lower the poverty rate. Therefore the country appears to be locked in a poverty trap.

*1.2. Quality of education and social groups dynamics*

According to the previous evidence, it seems that the Peruvian voters can be divided into three social groups:

- In the first one, "the lower class", people's human capital is below the poverty threshold $h_c$. They support a low tax rate to finance public education.
- In the second group, "the upper class", people's human capital is sufficiently high[3] so that they are willing to invest in private education. Therefore,they support a minimal tax rate.

[3]If the productivity of all factors is similar in public and private schools, people who earns more than the average income would prefer private education.

| Depen Var. | Primary education completed | Public ed. expenditure priority | Private ed. expenditure priority | Real education expenditure | Education expenditure if loan |
|---|---|---|---|---|---|
| Method | Probit | IV | IV | OLS | OLS |
| Schooling ( | 0.093 (5.9) | - | - | - | - |
| Schooling (mo | 0.11 (6.2) | - | - | - | - |
| Schooling | - | 0.82 (3.3) | (ns) | 0.023 (2.5) | (ns) |
| University | - | 11.3 (4.2) | 8.9 (4.2) | 0.51 (4.4) | 0.25 (3.4) |
| ln(Income) | - | 2.98 (2.8) | (ns) | 0.25 (6.2) | $-0.32$ (10.4) |
| Quecha spoken | $-0.38$ (3.8) | (ns) | 5.0 (2.7) | (ns) | (ns) |
| Urban area | $-0.16$ (2.6) | (ns) | (ns) | 0.19 (2.5) | (ns) |
| Lima | (ns) | $-10.5$ (5.3) | $-5.5$ (3.0) | (ns) | 0.28 (4.8) |
| Repetitions | 0.313 (3.0) | - | - | - | - |
| Alimentation | - | $-0.46$ (11.7) | - | - | - |
| Age | - | (ns) | $-0.16$ (2.5) | 0.0097 (3.1) | (ns) |
| Currently stu | - | (ns) | 7.34 (3.6) | - | (ns) |
| Savings, resl | - | - | $-0.24$ (8.7) | - | - |
| Children | - | (ns) | (ns) | $-0.13$ (5.5) | (ns) |
| Individual (nu | 1297 | 1523 | 1550 | 759 | 1379 |
| $R^2$ (or pseudo) | 0.29 | 0.14 | 0.10 | 0.26 | 0.07 |

Table 3: Impact of education on several indicators of education demand

- In the third group, "the medium class", people support high tax rate for a high quality of public education.

Basically the electoral output will depend of the relative size of these three groups. A government supporting higher taxes cannot emerge if the medium class does not weight more than half of the electoral body. To favor public education, politics will have to reduce poverty, without increasing too much the inequalities. Promoting private education can affect the political support for a better public education in two opposite ways:

- On the one hand, private education allows to increase the total educational expenditure, when public education quality is low. This improves human capital accumulation and reduces the size of the "lower class".
- But on the other hand, it may increase the number of agents who support a low tax rate because they prefer private education.

Section 2 presents the theoretical model. Section 3 determines the conditions of the political equilibrium. Section 4 considers the stationary distribution of human capital and section 5 exhibits the conditions of stability and uniqueness of the distribution allowing a high quality public education in the long-run.

## 2. The model

### *2.1. Main assumptions of the framework*

The following model aims to describe human capital dynamics in Peru in the recent decades. It relies on five major assumptions.

- The economy is made up of an infinity of dynasties $i$ whose human capital varies at each generation $t$. The dynasty member can produce a final consumption good within household firm or work as a teacher. Human capital is supposed to be the only factor of productivity and the labor market is balanced. Therefore, the income of the dynasty $i$at the generation $t$ $y_t^i$, is proportional to its human capital $h_t^i$.

$$y_t^i = w(\cdot) h_t^i \tag{1}$$

  This view is consistent with the human capital theory and is supported by the empirical evidences presented in the next section.

- Human capital of a member of the generation $t$ depends of the human capital of his parents and of school efficiency $E_t^i$ and of a idyosincratic shock $\theta_t^i$ This stochastic parameter represents the pure ability of the child. It is assumed independent of all other educational inputs and follows a log-normal law. The standard deviation of $\ln\theta$ is denoted $\omega$ and its expectancy is assumed to be zero without loss of generality.The law of motion of human capital within a dynasty $i$ at the generation $t$ has therefore the following form:

$$h_{t+1}^i = \theta_t^i \phi\left(h_t^i, E_t^i\right) \tag{2}$$

- The current level of public investment in education in Peru $D_t^q$ is assumed to rely on a political equilibrium. A flat tax, whose rate is $\tau_t$ and determined by a majority vote allows to finance public education. The expenditure depends of the aggregate human capital $H_t$ and the human capital distribution in the economy $G_t(h)$.

$$D_t^q = \tau_t Y_t = \tau\left(G_t\right) w_t H_t \tag{3}$$

- In this paper, a mix system of education is considered: educational expenditure can be private or public. It is assumed that both kind of expenditure cannot be cumulated because pupils cannot attend simultaneously private and public schools. This assumption is not fully consistent with the reality because people may choose to enter in a private university after a scholarship in public secondary school and conversely. An alternative would have been to consider that households investing in private education can benefit from part or all of the public expenditure they deserve[4]. Simulations have been conducted in these kind of model too and show that the results presented in this paper hold[5]. "Productivity"[6] of public and private schools may differ[7]. The relative productivity of private schools is noted $p$ and is supposed to be constant over time. At each time a share $n_t$ of children attend a public school. This parameter will be referred as the "size" of the public system in the followings. A pupil attending a public school benefit from a school

[4]This is typically the case when education vouchers are introduced or when quotas are imposed on private schools.

[5]A study of the political equilibrium and simulations results can be provided by the author.

[6]Productivity stands in the followings for the productivity of education after taking into account all inputs.

[7]According to many authors, private education seems to outperform public education, especially in developing countries and even after a relevant correction of the bias, see Psacharopoulos (1987), Halsey, Heath and Ridge (1980), Jimenez (1991), Williams and Carpenter (1991), Govinda and Varghese (1993).

efficiency $E_i^q$ equals to the real expenditure per pupil:

$$E_i^q = \frac{D_t^q}{w_t n_t} = \frac{\tau\left(G_t\right) H_t}{n_t} \tag{4}$$

The school efficiency $E_i^p$ of the private schools depends of the parental share of income devoted to private education $e_t$.

$$E_i^p = p_t e_t h_t \tag{5}$$

- Eventually it is assumed that the agent's behaviour reflects his own preference for education. Their private education expenditure as his vote derive from the maximisation of utility. The utility function $U$ is supposed to depend of his human capital $h_t$, his current consumption $c_t$ and future income of his heir $y_{t+1}$. Preferences of the households are heterogeneous and defined by an altruistic parameter $\beta\left(h\right)$. A joy of giving kind altruism is considered. In a more general frame, the agents should consider the utility of their child.but this specification lead to an infinite horizon maximization problem. This complicates the calculation without modifying the properties of the model, as long as the agent does not consider the endogeneity of preferences.

$$U(c_t, y_{t+1}) = u(c_t) + \beta(h_t)E\left[u\left(y_{t+1}\right)\right] \tag{6}$$

A logarithmic utility is used here for reason of simplicity, as in Glomm and Ravikumar's model. More complicated forms prevent to derive analytically the political equilibrium. A logarithmic utility allows indeed simplifications by making the stochastic term and the factor productivity $w$ disappear:

$$U(c_t, y_{t+1}) \equiv \ln\left(c_t\right) + \beta(h_t)\ln\left(\phi\left(h_t^i, E_t^i\right)\right) \tag{7}$$

These assumptions although simplistic are classic in the political choice theory. As $\theta$ is continuous, the distribution of human capital will be continuous too. This feature prevents a full analytical study of the the model. However discrete probabilities induce discontinuities making many thresholds values, always difficult to calibrate, to emerge. Assuming continuous probabilities although it requires simulations allows to reduce the need of calibration.

This model is then close to the Glomm and Ravikumar's one (1992) although preference for education, measured by the parameter $\beta$ is not homogenous among the population. Perotti (1993), Epple and Romano (1996), Fernandez and Rogerson (1996) or Desdoigt and Moizeau (2005) treat similar situations in political economy models where preferences are heterogeneous. But contrary to theses previous studies, social classes are not presupposed here. Using a continuum of human capital levels allows to define social class whose size are endogenous.

Using EVEP empirical evidences, it is assumed that $\beta$ takes only two different values depending on whether the agent's human capital is below or above the threshold level $h_c$.

$$\beta(h) = \begin{cases} \beta_L & if \quad h < h_c \\ \beta_H & if \quad h \geq h_c \end{cases} ; \beta_L < \beta_H \tag{8}$$

In this model, the impact of endogenous fertility on human capital accumulation is neglected. Fertility is well known to play an important role on education attainment and conversely education of women is supposed to decrease fertility. It is supposed nevertheless that the difference in the parameters $\beta$ accounts already for the links between education and fertility too. The population growth rate is assumed to be invariant among household and assumed to be null without loss of generality.

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Each agent lives two periods. An agent born in $t-1$ belongs to the generation $t$. In the first period, he goes to a public or private school. He gives birth to a single child at $t$, at the beginning of the second period. Then, he votes for a tax rate $\tau$ in order to finance the public education system. After that, he decides whether or not he will send his child to a private school and if he does, he chooses the share of his income he will devote to private education. He works and gets an income $y_t$ depending of its human capital. The distribution of human capital in the generation $t$, denoted by the cumulative density $G_t(h)$ completely defines the state of the economy during the period from $t$ to $t+1$.

*2.2. Empirical findings on human capital accumulation in Peru*

There are mainly two types of educational expenditure: infrastructures and compensations of teachers. In the long-run, infrastructure costs always represent a very limited share of the overall educational expenditure. Moreover, infrastructure costs are fixed costs because they do not vary with school time, qualifications of teachers and barely with teacher per pupil ratios. Although these costs may be lowered by technological progress, their marginal productivity tends toward zero when they are sufficiently high to guarantee a decent quality of infrastructures. It is true that in a developing country like Peru, the quality of school infrastructures may be insufficient especially in rural area. Nevertheless infrastructures productivity is heavily increasing with others inputs so that a lack of infrastructure is rather a consequence of a lack of human capital in the educational system than the opposite. Therefore, these costs will be neglected to ease the calculations.

Compensations of teachers are obviously proportional to hours of schooling $\nu$. Moreover, in a general equilibrium framework, teachers are paid regarding to their marginal productivity so that the payroll is also proportional to the teacher's human capital $H^T$. These expenditure are also inversely proportional to pupil/teacher ratio $l$. The marginal educational expenditure by child are:

$$D = \frac{\nu w H^T}{l} \tag{9}$$

Glomm and Ravikumar consider parental human capital $h_{t-1}$, school time $\nu$ and real educational expenditure per pupil $D$ as substitues, introducing $Q$ as the productivity of education[8].

$$h_{t+1} = Q\theta_t \nu^{\zeta} h_t^{\gamma} D_t^{\delta} \tag{10}$$

According to the litterature, a general equation should integrate hours of schooling, pupil/teacher ratio $l$, human capital of the teacher $H^T$ and peer effect through human capital of the other pupils $h^o$. When a log-linear form is assumed, it gives:

$$h_{t+1} = Q\theta_t h_t^{\gamma} v_t^{\zeta} \left(H_t^P\right)^{\eta} l_t^{-\varphi} \left(h_t^o\right)^{\xi} \tag{11}$$

The human capital of the peer can be expressed as a geometric average of the parental human capital and the average human capital, so that the parameter $\xi$ can be set to zero without loss of generality. Pluging the accountable relation **(NN)** into the previous one gives:

$$h_{t+1} = Q\theta_t h_t^{\gamma} v^{\zeta-\varphi} \left(H_t^P\right)^{\eta-\varphi} \left(\frac{D}{w}\right)^{\varphi} \tag{12}$$

Microeconomic data strengthen than there are more hours of schooling in private schools. However, it is obvisouly possible because expenditure per pupil are higher. The same idea applies for teachers' qualification. As neither school time nor teachers' qualifications cannot be observed in the data, it is assumed here that they both increase with the real expenditure $D/w$.The following simplified form is introduced when the parameter $\delta$ is supposed to measure both direct and indirect effects (that is better teachers and more hours of schooling) of educational expenditure.

$$h_{t+1} = Q\theta_t h_t^{\gamma} E_t^{\delta}; E_t = \{E_t^p \text{ or } E_t^q\} \tag{13}$$

Using these features, human capital accumulation is independent of labor productivity and then physical accumulation. Global return to scale are supposed to be decreasing, $\gamma + \delta < 1$, because human capital shows large diminishing returns

[8] Which corresponds to educational technology, that may have increased with discoveries easing knowledge accumulation such as writing, printing and computer science for instance.

among time[9] and this hypothesis is consistent with our estimations on Peruvian data.

Estimates of the equation **(NN)** is made in two steps. First, impact of parental education is estimated by regressing schooling on parents' schooling (equation **NN**). It appears that the type of school, private or public, or the fees have no impact on years of schooling. This regression allows to estimate that the parameter $\gamma$ is around 0.4. Years of schooling is $h$ for the individual and $h_f$ and $h_m$ stand for years of schooling of his father and mother.

$$h = \underset{(6.9)}{0.22}h_f + \underset{(6.1)}{0.20}h_m + \underset{(6.0)}{1.85}urban\_area + \underset{(62.6)}{0.68}Lima + \underset{(31.2)}{6.88} \quad (14)$$

$N = 1327$, adjusted $R^2 = 0.22$, mothertongue and gender dummies are unsignificant.

In Peru, private education represents a significative part of the educational supply, about 10.3% of the labour force according to the EVEP survey. The quality of private education, measured by education returns differs only from public education due to a higher level of expenditure. There are no specific positive effect of the private education sector on the education returns according to EVEP data. This finding indicates that the relative productivity factor of private education $p \simeq 1$. To estimate the impact of educational expenditure on education returns, the logarithm of fees is introduced in a standard mincerian regression. This regression is run only when the contestant or his spouse is the head of the household in order to limit endogeneity problems. The expenditure is supposed to be equal to 7 soles per month in public schools[10]. This regression allows to determine an upper bound for the parameter $\delta$, as every endogeneity problems are not adressed here, which is about 0.2. $h_h$ stands for the schooling of the head of household, $L$ for the number of workers in the household and $E_h$ for the educational expenditure declared to have been received by the head of household.

$$\ln y = \underset{(9.1)}{0.07}h_h + \underset{(3.3)}{0.04}age_h - \underset{(2.4)}{0.0003}age_h^2 + \underset{(3.9)}{0.26}\ln L + \underset{(3.8)}{0.38}urban\_area + \underset{(10.7)}{0.83}Lima + \underset{(4.6)}{0.24}\ln E_h + \underset{(11.2)}{3.3} \quad (15)$$

$N = 601$, adjusted $R^2 = 0.39$, mothertongue, gender and private education dummies are unsignificant.

In the followings this framework is used to derive the optimal behaviour of the

[9] Here we consider the social returns. Evidences support indeed increasing private returns of education at the microeconomic level (UNESCO 2003). However, the elasticity of productivity to aggregated human capital seems not to be very large when evaluated by cross-country analysis.

[10] This figure has been calculated to be comparable with level of expenditure in private schools.

agents.

## 3. The political equilibrium

To study the political equilibrium, the first step is to determine the private education expenditure, for a given taxe rate $\tau$.

### *3.1. Equilibrium on the private education market*

After the vote of the tax rate, the decisions of households regarding private education is determined by the relative efficiency of the public and the private schools. The quality of private schools is exogenous and the quality of public schools only depends of the size of the public system $n$. The equilibrium on the private education market is reached when the efficiency of public schools, is such that no household can improve its utility by modifying its decision regarding private education. As the decisions on education depends of the anticipated quality of education in the public sector, at the equilibrium, the expectations of househoulds regarding private education are rationnal. The first proposition proves that for a given tax rate, such an equilibrium is unique. To simplify the calculations, let us introduce the "preference" for education $\rho_h$, which takes two values $\tau_L$ and $\tau_H$ such that $\tau_i = \frac{\beta_i \delta}{1+\beta_i \delta}$ for $i = \{L, H\}$ whether the agent is below the poverty line or not.

$$\rho_h = \begin{cases} \tau_L & if \quad h < h_c \\ \tau_H & if \quad h \geq h_c \end{cases} ; \tau_L < \tau_H \tag{16}$$

**Proposition 1.** *For a given tax rate $\tau$ and a given distribution of human capital $G$, there is a unique threshold level of human capital $h_s$ such that an agent with human capital $h$ send his child to a private school if and only if $h > h_s$. In this case, such an agent invests a share $\rho_h (1-\tau)$ of his income in private education. Consequently, the size of the public system $n$ is a well defined function $N(\tau, G)$ of the tax rate and the distribution. Hence, for each agent and for $n$ given, the utility is a continuous function of $\tau$, $u(\tau) = \max(u^r(\tau), u^q(\tau))$ :*

$$\begin{array}{rc} u^q(\tau) = & \ln(1-\tau) + \beta(h)\delta \ln\left(\tau \frac{H}{n}\right) \\ u^r(\tau) = & \ln\left(\frac{1-\tau}{1+\beta(h)\delta}\right) + \beta(h)\delta \ln(\mu \rho_h (1-\tau) h) \end{array} \tag{17}$$

See the appendix for a detailed proof. A simple study of the utility function $u(\tau)$ shows that preferences are not single-peaked (see figure 2). Therefore, the median voter theorem can not be applied here and the existence of a tax rate which is a Condorcet winner should be studied "manually". It is considered here that the voters determine their political choice considering the quality of the public education system as given: they do not anticipate the consequences of the vote output on the size of the public system $n$ and therefore the quality of the education. This assumption is neither sufficient nor necessary to garantee the existence of a political equilibrium. However, although it eases heavily the calculations, this assumption is likely. The quality of education is difficult to anticipate as the distribution of the human capital is probably not perfectly known by the voters. After the vote however, the agents can observe the quality of both public and private school and the private education market can converge to an equilibrium. **fhFU7.6882cm5.0918cm0ptVariation of utility with tax rate for a typical agent.utility**$_s tandard.tif$

**If there is a political equilibrium, there is a tax rate which is a Condorcet winner. To be a Condorcet winner, a tax rate should be preferred over all the other by more than half of the voters. In what follows, $\tau \succ \tau'$ means that there is a majority of voters who prefer the tax rate $\tau$ over $\tau'$.**

**Proposition 2.** ***The only tax rates which can be Condorcet winners are*** $\{0, \tau_L, \tau_H\}$

**Proof** **Let us suppose $\tau^* > 0$ a Condorcet winner. Thus for an agent belonging to the majority, and for any $\tau \neq \tau^*$ :**

$$\max\left(u^q\left(\tau^*\right), u^r\left(\tau^*\right)\right) > \max\left(u^q\left(\tau\right), u^r\left(\tau\right)\right) \tag{18}$$

**As $u^r\left(\tau^*\right) < u^r\left(0\right)$, it leads: $\max\left(u^q\left(\tau^*\right), u^r\left(0\right)\right) > u^q\left(\tau^*\right)$. But as $\tau^* > 0$ is preferred by the member of the majority, $u^q\left(\tau^*\right) > u^r\left(0\right)$ and the condition becomes $u^q\left(\tau^*\right) > u^q\left(\tau\right)$. Therefore the only possible strictly positive values of $\tau^*$ are the maxima of $u^q\left(\tau\right)$, which is $\tau_L$ for the poor agents and $\tau_H$ for the others.** □

**To determine if there is a political equilibrium, the three potential tax rate 0, $\tau_L$ and $\tau_H$ should be compared two by two in term of political support. The following property establishes the condition the distribution has to verify so that there is a Condorcet winner and consequently a political equilibrium.**

**Proposition 3.** ***For an exogenous anticipated size of the public education $n^a$ and for a given distribution of human capital $G$, there are well defined threshold values of h, $h_p$¡$h_m$¡$h_a$¡$h_r$ such that:***

**(i)** $\tau_H \succ \tau_L \Leftrightarrow s_{HL} = \max\left(G\left(h_r\right) - G\left(h_c\right), 0\right) > \frac{1}{2}$

**(ii)** $\tau_L \succ 0 \Leftrightarrow s_{L0} = \max\left(\min\left(G\left(h_a, h_c\right), G\left(h_m\right)\right)\right) > \frac{1}{2}$

**(iii)** $\tau_H \succ 0 \Leftrightarrow s_{H0} = \min\left(G\left(h_p\right), G\left(h_c\right)\right) + \max\left(G\left(h_a\right) - G\left(h_c\right)\right) > \frac{1}{2}$

***There is a Condorcet winner $\tau^*$ and then a political equilibrium if and only if $\tau^* \succ \tau$ for any $\tau \in \{0, \tau_L, \tau_H\} - \tau^*$.***

**Proof**

**(i) $\tau_H$ vs. $\tau_L$. Agents below the poverty line $h_c$ prefer taulow. Other agents vote for $\tau_H$ if and only if $u^q\left(\tau_H\right) > u^r\left(\tau_L\right)$. This condition leads to $h > \frac{H}{pn^a}\left(1-\tau_L\right)^{-1-1/(\delta\beta_L)} \equiv h_f$. $\tau_H$ is then supported by people whose human capital is above $h_c$ and below $h_f$.**

**(ii) $\tau_L$ vs. 0. As $u^r\left(0\right) > u^r\left(\tau_L\right)$, agents vote for $\tau_L$ if and only if $u^q\left(\tau_L\right) > u^r\left(0\right)$. This leads to $h > \frac{H}{pn^a} = h_a$ for agents below the poverty line and to $h > \frac{H}{pn^a}\frac{\tau_L}{\tau_H}\left(\frac{1-\tau_L}{1-\tau_H}\right)^{1/(\delta\beta_H)} = h_m$ for agents above the poverty line. Therefore, agents supporting $\tau_L$ are $\min\left(G\left(h_c\right), G\left(h_a\right)\right) + \max\left(G\left(h_m\right) - G\left(h_c\right), 0\right)$.**

**(iii) $\tau_H$ vs. 0. Agents vote for $\tau_L$ if and only if $u^q\left(\tau_H\right) > u^r\left(0\right)$.This leads to $h > \frac{H}{pn^a}\frac{\tau_H}{\tau_L}\left(\frac{1-\tau_H}{1-\tau_L}\right)^{1/(\delta\beta_L)} = h_p$ for agents below the poverty line and to $h > \frac{H}{pn^a} = h_a$ for agents above the poverty line. Therefore, agents supporting $\tau_L$ are $\min\left(G\left(h_p\right), G\left(h_c\right)\right) + \max\left(G\left(h_a\right) - G\left(h_c\right)\right)$.**

□

**As a consequence of this proposition, there is not always a political equilibrium. Although this could cause political instability in the short run, the stability in the long-run depends whether the previous proposition is verified for the potential stationary distributions. This proposition allows to define the map $\mathcal{T}\left(G, n^a\right)$ which links to any distribution the value of the tax rate which is a Condorcet winner if there is one and $-1$ otherwise.**

## 4. Stationary distributions

**Introducing $\Xi_t = H_t/n_t$, human capital dynamics in an educational system where agents have to choose between private or public education is described by the following stochastic process $\Psi$ :**

$$h_{t+1} = \Psi\left(\theta_t, h_t, \tau_t, \Xi_t\right) = \begin{cases} Q\theta_t h_t^\gamma \left(\Xi_t \tau_t\right)^\delta & if \quad h \leq h_s\left(\Xi_t\right) \\ Q\theta_t h_t^\gamma \left(p\rho\left(h\right)\left(1-\tau\right)h\right)^\delta & if \quad h > h_s\left(\Xi_t\right) \end{cases} \tag{19}$$

**This process is not Markovian because the dynamic of a dynasty depends of external variables $\Xi_t$ and $\tau_t$. Consequently, the stationary distributions of the process $\Psi$ cannot be directly determined. To solve this problem, stationary distributions will be studied for frozen values of parameters $\Xi$ and $\tau$.**

*4.1. Existence and uniqueness when tax rate and mean income remain constant*

**Until here, human capital $h$ and stochastic parameter $\theta$ where defined on non compact sets. This approach does not permit neither to run numericial simulations of the process $\Psi$ nor to apply theorems of existence and uniqueness about stationary distributions of Markovian processes. To ease the resolution, the problem is restricted to a compact set $X \times Z$ such that:**

$$X \equiv [\underline{h}, \overline{h}], Z \equiv [\underline{\theta}, \overline{\theta}] \tag{20}$$

**The dynamic of human capital depends of two agregated variables, the ratio $\Xi = H/n$ and the tax rate $\tau$. Assuming that $G$ is such that there is a political equilibrium, the stochastic process $\Psi$ becomes Markovian when $\Xi$ and $\tau$ are assumed to remain constant. The following property presents the conditions for the existence of a stationary distribution when the efficiency of public education $\Xi$ and the tax rate $\tau$ are assumed exogenous and constant over time.**

**Proposition 4.** ***Assuming that :***

1. ***There are a smallest $\underline{h}$ and a largest $\bar{h}$ reachable level of human capital.***
2. ***There are a smallest $\underline{\theta}$ and a largest $\bar{\theta}$ ability factor.***
3. ***The human capital accumulation obeys to the Markov process whose transition function is $\hat{\Psi}_{X,Z}$ , defined on $X \times Z \to X$ :***

$$\hat{\Psi}_{X,Z}(h,\theta,\Xi,\tau) = \begin{cases} \underline{h} & \textbf{\textit{if}} \quad \Psi(h,\theta) < \underline{h} \\ \Psi(h,\theta,\Xi,\tau) & \textbf{\textit{if}} \quad \Psi(h,\theta,\Xi,\tau) \in X \\ \overline{h} & \textbf{\textit{if}} \quad \Psi(h,\theta,\Xi,\tau) > \overline{h} \end{cases} \tag{21}$$

4. ***$\theta_t$ has the following "modified" log-normal transition law $Q_Z$:***

$$Q_Z(\theta,[b,b+db]) = \begin{cases} \frac{d\Lambda_{\mu,\sigma}(b)}{\Lambda(\bar{\theta})-\Lambda(\underline{\theta})} & if \quad b \in Z \\ 0 & otherwise \end{cases} \tag{22}$$

***where $\Lambda_{\mu,\sigma}$ is the cumulative density of the log-normal distribution with associated mean $\mu$ and standard deviation $\sigma$.***

***Then the Markovian process defined by*** $h_{t+1} = \hat{\Psi}_{X,Z}(h_t, \theta_t, \Xi, \tau)$ ***admits a stationary distribution, which is unique if*** $X$ ***and*** $Z$ ***are large enough.***

**A proof can be found in the appendix. The needed conditions are not restrictive.**

*4.2. A condition for stationary distribution*

**Now that we have proved the existence and uniqueness of the stationary distribution, we can define the map** $\mathcal{G}(\Xi, \tau)$ **which associates to the couple** $\{\Xi, \tau\}$**the unique stationary distribution of the process** $\hat{\Psi}_{X,Z}(h, \theta, \Xi, \tau)$. **Moreover the following proposition shows that for a given** $\tau$ **there is a unique stationary distribution for the process** $\hat{\Psi}_{X,Z}(h, \theta, \Xi, \tau)$ **where** $\Xi = H/N(G, \tau)$**.**

**Proposition 5.** ***For a given*** $\tau$***, there is a single*** $\Xi^*(\tau)$ ***such that for the unique stationary distribution*** $G = \mathcal{G}(\Xi^*, \tau)$, $\Xi^* = \int h dG(h)/n(G, \tau)$.

**The proof is available in appendix. The previous proposition allows to define the map** $\mathcal{H}(G, \tau) = \int h dG(h)/N(G, \tau)$. **The following property will be used to obtain the stationary distributions in the general case.**

**Proposition 6.** ***Considering the same restrictions as in proposition 4, the distribution*** $G$ ***is a stationary one for the process*** $\hat{\Psi}_{X,Z}(h_t, \theta_t, \Xi_t, \tau_t)$ ***if and only if there exists a couple of real*** $\{\Xi, \tau\}, 0 \le \tau < 1$ ***such that:***

$$\begin{cases} \mathcal{H}(G, \tau) - \Xi = 0 \\ \mathcal{T}(G, N(G, \tau)) - \tau = 0 \end{cases} \tag{23}$$

$$with \quad G = \mathcal{G}(\Xi, \tau) \tag{24}$$

**Proof** **Suppose that** $G$ **is a stationary distribution for** $\hat{\Psi}(h_t, \theta_t, \Xi_t, \tau_t)$**. Thus** $\mathcal{T}(G, N(G, \tau_{t-1})) = \tau_t$ **and there exists** $\tau$ **such that** $\mathcal{T}(G, N(G, \tau)) = \tau$**. Let us denote** $\Xi = \mathcal{H}(G, \tau)$**. By definition,** $\tau$ **and** $\Xi$ **verifies (NN). As** $G$ **is a stationary distribution for** $\hat{\Psi}_{X,Z}(\bullet, \Xi_t, \tau_t)$**, at the steady state,** $\tau_t = \tau$ **and** $\Xi_t = \mathcal{H}(G, \tau) = \Xi$**. Consequently,** $\hat{\Psi}_{X,Z}(\bullet, \Xi_t, \tau_t) = \hat{\Psi}_{X,Z}(\bullet, \Xi, \tau)$. **Since the process** $\hat{\Psi}_{X,Z}(\bullet, \Xi, \tau)$ **converges to a unique stationary distribution, it implies that** $\mathcal{G}(\Xi, \tau) = G$. □

**These two propositions show that for a given tax rate, there is only one stationary distribution, which is such that** $\mathcal{H}(\mathcal{G}(\Xi^*, \tau)) = \Xi^*$**. For a** $\tau$ **given, the parameter** $\Xi^*(\tau)$ **can be determined using numerical simulations of the stationary distributions** $\mathcal{G}(\Xi, \tau)$ **and dichotomic algorithms.**

As there are only three possible candidate for the stationary tax rate, there are only three possible stationary distributions, which can be noted $G_i, i = \{0, L, H\}$ such that

$$G_i = \mathcal{G}\left(\Xi^*\left(\tau_i\right), \tau_i\right), i = \{0, \tau_L, \tau_H\} \tag{25}$$

These distributions can be considered as "stable", considering the political equilibrium if the tax rate $\tau_i$ is the Condorcet Winner for the stationnary distribution $G_i$. In the following section, we use numerical simulations of the stationary distributions to determine the conditions the parameters $Q$ and $p$ have to verify so that the stationary distributions would be stable considering the political equilibrium.

## 5. The development trap

To assess the impact of private education on the stationary distribution of human capital, let us consider first the case of a pure public system. In such a framework, there is no private education. The accumulation process becomes easier and log-linear:

$$h_{t+1} = Q\theta_t h_t^{\gamma}\left(\tau H_t\right)^{\delta} \tag{26}$$

There are only two possible values for the tax rate $\tau_i$, $\tau_L$ and $\tau_H$ and two possible stationary distributions, which are log-normal with associated standard deviation $\sigma$ and median $\mu_i$

$$G_i = \Lambda_{\mu_i,\sigma}; \begin{cases} \mu_i = \frac{\ln Q + \delta\left(\ln \tau_i + \frac{\sigma^2}{2}\right)}{1-\gamma-\delta} \\ \sigma = \omega/\sqrt{1-\gamma^2} \end{cases} \tag{27}$$

In such a system, the preference are single-peaked, the median voter theorem can be applied and $\tau = \tau_L \Leftrightarrow G\left(h_c\right) > \frac{1}{2}$. Therefore, the "low" stationary distribution, which is associated to the lower tax rate $\tau_L$ is possible if and only if $\mu_L < \ln\left(h_c\right)$. Symmetrically the "high" stationary distribution, which is associated with the higher tax rate is possible if and only if $\mu_H > \ln\left(h_c\right)$. Hence there are three cases:

(i) If $Q < \underline{Q} \equiv h_c^{1-\gamma-\delta}\tau_H^{-\delta}e^{-\frac{\delta}{2}\sigma^2}$, the mean is always below the poverty line: the "high" stationary distribution is never reachable and the economy will never be able to adopt a high quality public system.

(ii) If $Q < \underline{Q} \equiv h_c^{1-\gamma-\delta}\tau_H^{-\delta}e^{-\frac{\delta}{2}\sigma^2}$ the mean is on the contrary always above the poverty line. As $\overline{Q} > \underline{Q}$, there is therefore only one stationary distribution and the economy cannot be trapped in poverty.

(iii) Eventually, for $Q \in ]\underline{Q}, \overline{Q}[$ both stationary distributions are possible and the economy can be locked in a poverty trap if the initial stationary distribution is too close to the lowest one.

In the following, it is supposed that $Q \in ]\underline{Q}, \overline{Q}[$, otherwise there would be no need for any improvment of the educational system. It is also supposed that the relative efficiency of the private sector parameter $p$ is unknow but in the range $[0.2, 1.6]$. According to EVEP rough data, it is indeed likely that $p$ is about one.

To determine the stationary distributions of the model, numerical simulations of $\mathcal{G}(\Xi^*(\tau_i), \tau_i)$ are calculated on the grid $[\underline{Q}, \overline{Q}] \times [0.2, 1.6]$ for the couple of parameters $Q \times p$[11].To do the numerical simulations, the parameters of the model are calibrated using the EVEP data: $\delta = 0.2$, $\gamma = 0.4$, $\omega = 0.5$. The preference parameters $\tau_L$ and $\tau_H$ can not be directly deduced from the microeconomic data. Simulations use $\tau_L = 0.03$, about the average value of the public expenditure in education as a share of GDP in contemporary Peru, and $\tau_H = 0,07$, the approximate value of public expenditure in developed countries according to UNESCO data. Nevertheless, all the following results hold for other values of the parameters as long as they remain realistic.

**Proposition 7.** ***For the calibrated values of the parameters*** $\delta = 0.2$, $\gamma = 0.4$, $\omega = 0.5$***, there is, for a given value of*** $Q \in [\underline{Q}, \overline{Q}]$***, a maximal value of*** $p^*$ ***such that the null tax rate is not a Condorcet winner for*** $p < p^*$ ***in the long run in a pure private system.***

fhFU12.1166cm7.6838cm0ptCurves $s_{L0}$ and $s_{H0}$ constant and values of $\{p, Q\}$ such that a privatized educational system is stable.private$_{s}$$tability.tifOnthefigure$
$s_i$, the sets of parameters $\{Q, p\}$ such that there is a share $s_i$ of the voters who prefer the tax rate $\tau_j$ over a privatized system. For a given $Q$, the simulations show that the functions $s_{L0}(Q, p)$ and $s_{H0}(Q, p)$ are decreasing with $p$. A pure private system is stable if and only if a null tax rate is a Condorcet winner when the distribution of human capital equals the unique[12] stationary distribution of a privatized system. The conditions $s_{L0}(Q, p) < \frac{1}{2}$ and $s_{H0}(Q, p) < \frac{1}{2}$ should both be verified. The sets of

[11] More details about numerical simulations can be provided by the author if requested.

[12] In a privatized system, there are no externalities in the accumulation process, which is thus Markovian. Hence, proposition directly holds.

**parameters $\{Q, p\}$ such that a privatized system is stable corresponds to the hatched area on the figure (FF). Simulations shows that for values of the parameter $p$ below unity, the pure private system is never stable. However, for high levels of educational efficiency $Q$, a privatized system is stable if the productivity of education is higher in private school. Introducing private education in an economy may lead to the full privatization of the educational system in the long-run if the productivity of the private schools are higher. However, there are few reasons to believe that in the long-run, private schools productivity would be much higher than public schools one, after taking into account peer effects[13] and difference in the level of expenditure. Decentralization and social control of the education workers may indeed be implemented in publicly funded schools too. This property is consistent with the empirical fact that all over the world there is only few countries, in which public school do not exist.**

**To assess the long-run impact of private education on the public investment in human capital, the functions $s_{ij}$ which expresses the share of the voters which prefers the tax rate $\tau_i$ over the tax rate $\tau_j$, are calculated for simulated stationary distributions on the grid. The following property indicates that introducing a private system of education is usefull to reduce the size of the poverty trap, which can be seen as the set of values of $Q$ for which several stationary distributions are possible.**

**Proposition 8.** ***For the calibrated values of the parameters*** $\delta = 0.2, \gamma = 0.4, \omega = 0.5$***, there is a strictly positive value of*** $p$ ***which allows to minizes the space***$[\underline{Q}_p, \overline{Q}_p]$***, so that there are not only one stationary distribution for*** $Q \in [\underline{Q}_p, \overline{Q}_p]$***.***

**fhFU12.1166cm7.4136cm0ptCurves $s_{ij} = \frac{1}{2}$ for different tax rate $\tau_L$ and $\tau_H$ and values of $\{Q, p\}$ such that the high distribution is stable.high**$_{s}tability.tifThecur$
**$\frac{1}{2}$ are calculated for the two stationary distributions with $\tau = \tau_L$ or $\tau_H$. Below the highest of these two curves, the higher tax rate is not a Condorcet winner in the long run. Therefore, the higher stationary distribution is unstable. The curves $s_{H0} = \frac{1}{2}$ are also calculated for the two possible stationary distributions with non nul tax rate. When $\tau = \tau_L$, the high tax rate $\tau_H$ is always preferred by a majority of voters for the parameters $\{Q, p\} \in [\underline{Q}, \overline{Q}] \times [0.2, 1.2]$ and is therefore not represented on**

[13]Peer effects are not modelled here but introducing them in the human capital accumulation function does not modify the form of the problem and have no impact on the results.

the figure. However, when $\tau = \tau_H$, the stationary distribution is such that a privatized system ($\tau = 0$) is preferred over a high tax rate ($\tau_H$) by a majority of voters for values of $p$ above 1,1. The high tax rate is a Condorcet winner if the parameters $\{Q, p\}$ belongs to the blank area.

fhFU10.1067cm6.1901cm0ptValues of $\{Q, p\}$ such that the high stationary distribution is the only one.property9.tif

When the parameters $\{Q, p\}$ are such that the high quality education system is stable and the privatized system is not, the high tax rate $\tau_H$ is the only value possible of the tax rate in the long-run. When the couple $\{Q, p\}$ belongs to the white area in the above figure, introducing private education in a mix system allows to make the poverty trap disappear. On the contrary when the couple $\{Q, p\}$ belongs to the hateched area on the left, when the relative productivity of private schools is high, the economy will surely adopt a privatized system in the long-run.

Proposition 9. ***Consequently, introducing a private system of education always better the quality of the public system in the long run when the productivity of factors is similar in the private and the public system.***

Proof When $P \approx Q$, a privatized system is not possible in the long run according to the simulations. Therefore, the tax rate $\tau \geq \tau_H$. Introducing private education in a public system thus allows to increase the quality of education of the richest agents without harming the quality of education in the public schools. For a given $h_t$ and a given average human capital in the economy $H_t$, the human capital of the child will therefore be higher in a mix system. As this is true for all agents, the average human capital is higher too in a mix system and the efficiency of education in public schools is also higer. □

## 6. Concluding remarks

This study provides an explanation for the persistence of low public investment in education in the poorest economies. When people below the poverty line exhibit a lower preference for education, as shown by microeconomic data in the case of Peru, a political majority supporting higher tax for higher expenditure in education may be impossible. As there are several stationary distributions possible, some poor economies can be trapped in underdevelopment. Nevertheless, this paper shows that introducing private education as an alternative to public one allows to lower poverty and increase the size of the middle class supporting higer expenditure for public schools. For reasonable values of the parameters, calibrated on Peruvian data, private schools can make the poverty trap disappear if the productivity of the public schools is sufficient.

Simulations indicate nevertheless that if the productivity of education (after taking into account all factors and peer effects) is higher in the private schools than in the public ones, a majority of voters supporting the total privatization of the system may appear. This scenario appears unlikely in Peru, as productivity of education in both sectors seems to be similar after taking into account differences in the level of expenditure.

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## Appendix

*Proof of property 1*

The second period budget constraint becomes:

$$c_t = w_t h_t (1 - \tau_t - e_t) \tag{28}$$

Combining equations (NN) and (NN), the utility can be simplified as:

$$U(e,\tau) = \begin{cases} \ln(1-\tau-e_t) + \beta\delta \ln(pe_t h_t) \equiv U^r(e,\tau) & if \quad e > 0 \\ \ln(1-\tau) + \ln\left(\tau \frac{H_t}{n_t}\right) \equiv u^q(\tau) & if \quad e = 0 \end{cases} \tag{29}$$

If the agent send his child to a private school, the FOC on $e$ gives the optimal education expenditure as:

$$e = \frac{\beta(h)\delta}{1+\beta(h)\delta}(1-\tau) \equiv \rho_h(1-\tau) \tag{30}$$

The utility of the agent which send his child to a private school can be expressed as a function of $\tau$ only.

$$U^r(\rho_h(1-\tau),\tau) = u^r(\tau) \equiv \ln((1-\tau)(1-\rho_h)) + \beta(h)\delta\ln(p\rho_h(1-\tau)h_t) \tag{31}$$

For a given $n$, the kind of school the child will attend depends on the sign of the expression $e > 0 \Leftrightarrow u^r(\tau) - u^q(\tau) > 0$. The function $u^r(\tau)$ is obviously decreasing and the function $u^q(\tau)$ is increasing on $[0, \rho_h]$ and decreasing after on $[\rho_h, 1[$. The utility of the agent varies with the level of tax $\tau$, $u(\tau) = \max(u^r(\tau), u^q(\tau))$This condition gives: $e > 0 \Leftrightarrow \ln(1-\rho_h) + \beta(h)\delta\ln(\mu\rho_h(1-\tau)h_t) - \ln\left(\tau\frac{H_t}{n_t}\right) > 0$. This allows to identify two threshold values $h_L$ and $h_H$:

$$e > 0 \Leftrightarrow h > \frac{H_t}{pn_t}\frac{\tau}{\rho_h(1-\tau)}(1-\rho_h)^{-1/(\beta(h)\delta)} \equiv h_i, i = \{L, H\} \tag{32}$$

A simple study shows that for any $n$, $G$ and $\tau$, $h_H < h_L$. Hence, for a given $n$, there is a unique threshold value $h_s = \max(\min(h_c, h_L), h_H)$ such that an agent invests in private education if and only if its human capital is above the threshold. The threshold values $h_i$ are increasing function of $\Xi$, as $h_s$. Hence, $h_s$ is a decreasing function of $n$. Thus the function $n - G(h_s(n))$ is strictly increasing in $n$. It tends toward a strictly positive value when $n$ tend toward 1. Therefore, there is only one value of $n$ such that $n - G(h_s(n)) = 0$. This unique value defines the equilibrium of the private education sector. It depends of the tax rate $\tau$ and the distribution $G$ and can be defined as a function of these variables, $N(\tau, G)$.

*Proof of property 4*

The function $\Psi$ is strictly increasing in $h$ and $\theta$ which implies that $\hat{\Psi}_{X,Z}$ is also increasing in those variables.

Let us define the space $S = X \times Z$ and $\mathcal{S}$ the product $\sigma$-algebra. We can now define $P$, a transition function for a Markov process as the mapping $P : S \times \mathcal{S} \rightarrow [0,1]$ defined for $A \times B$ elements of $S \times \mathcal{S}$ by:

$$P(h, \theta; A \times B) \begin{cases} Q_Z(\theta, B) & \text{if} \quad \hat{\Psi}_{X,Z}(h, \theta, Y, \tau) \in A \\ 0 & \text{otherwise} \end{cases}$$

The function $\hat{\Psi}_{X,Z}$ is measurable and increasing. The transition function $Q_Z$ of the random variable $\theta_t$ is independent of the position $\theta_t$, hence it is increasing. Lastly, $X$ and $Z$ are compact metric spaces endowed with closed orders (the usual order on $\mathbb{R}$) and with minimum elements. We have just verified all the hypothesis of the corollary 5 by Hopenhayn and Prescott (1992), which guarantees in this case that the Markov process $P$ has a stationary distribution.

One can define two parameters $q_L$ and $q_H$ such that

$$\Psi(h, \theta, \Xi, \tau) = \begin{cases} \theta h^{\gamma} q_L & \text{if} \quad h \leq h_s(\Xi) \\ \theta h^{\gamma+\delta} q_H & \text{if} \quad h > h_s(\Xi) \end{cases} \tag{33}$$

For given $\theta, \Xi$ and $\tau$, the function $\Psi$ has therefore two potential fix points, $(\theta q_L)^{1/(1-\gamma)}$ and $(\theta q_H)^{1/(1-\gamma-\delta)}$. However, for $\theta < h_s^{1-\gamma}/q_L = f_L$ or $\theta > h_s^{1-\gamma-\delta}/q_H = f_H$ $\Psi$ has only one fix point. Thus, for $\theta < \theta_L = \min(f_L, f_H)$ or $\theta > \theta_H = \max(f_L, f_H)$, the function $\Psi$ has only one fix point. Moreover, for $\theta < \theta_L$, the fix point verifies $\Sigma(\theta) < h_s$ and symetrically $\Sigma(\bullet) > h_s$ for $\theta > \theta_H$. Consequently, for any $\theta < \theta_L$ and $\theta' > \theta_H$, the function $\Psi$ has only one fix point $\Sigma(\theta)$ and $\Sigma(\theta) < \Sigma(\theta')$.

If $Z$ is sufficiently large, that is $\underline{\theta} < \theta_L$ and $\overline{\theta} > \theta_H$, for any $\theta \in ]\underline{\theta}, \theta_L]$ and for any $\theta' \in ]\theta_H, \overline{\theta}]$, $\Psi$ has only one fix point. If $X$ is sufficiently large too, that is $\underline{h} < \min\limits_{[\underline{\theta}, \theta_L]} \Sigma(\theta)$ and $\overline{h} > \max\limits_{[\theta_H, \overline{\theta}]} \Sigma(\theta)$, then, independently of the initial level of human capital $h_0 \in X$, we have for any $\theta \in [\underline{\theta}, \theta_L[$ and

$\theta' \in ]\theta_H, \overline{\theta}[$:

$$\underset{t\to\infty}{lim} h_t(\theta) = \underset{t\to\infty}{lim} \hat{\Psi}^t_{X,Z}(\theta, h_0) = h^*_L \tag{34}$$
$$\underset{t\to\infty}{lim} h_t(\theta') = \underset{t\to\infty}{lim} \hat{\Psi}^t_{X,Z}(\theta', h_0) = h^*_H$$

**Let us define $s = \frac{1}{2}(h^*_L + h^*_H)$. Due to the previous convergence properties, there are integers $n$ and $n'$ so that :**

$$\hat{\Psi}^n_{X,Z}\left(\theta, \overline{h}\right) < s \quad and \quad \hat{\Psi}^{n'}_{X,Z}\left(\theta', \underline{h}\right) > s \tag{35}$$

**As the transition function $Q_Z$ is independent of $\theta$, we have the probability**

$$\mathcal{P}(\theta_t = \theta, \forall t \leq n) = \mathcal{P}(\theta_t = \theta)^m > 0 \tag{36}$$

**Hence there is a point $s \in X$ and an integer $m = \max\{n, n'\}$ such that $P^m\left(\bar{h}, [\underline{h}, s]\right) > 0$ and $P^m\left(\underline{h}, [s, \bar{h}]\right) > 0$.**

**The Mixed Monotony Condition is verified, and the Theorem 2 of Hopenhayn and Prescott (1992) can be applied . There is a unique stationary distribution $G^*$ for the process $P$ and for any initial distribution $G_0$, $T^n G = \int P^n(s, \bullet)G(ds)$ converges to $G^*$.**

*Proof of property 5*

**Let us suppose that for any sequence of $(\theta_i)_{i=1}^T$ and for any $h_0$, $h_T = \Psi^T(h_0, \Xi)$ verifies:**

$$\frac{1}{h_T}\frac{dh_T}{d\Xi} < \frac{1}{\Xi} \tag{37}$$

**Using the definition of $\Psi$(NN), the property can be extended to the next rank:**

$$\frac{1}{h_{T+1}}\frac{dh_{T+1}}{d\Xi} = \begin{cases} \frac{\gamma}{h_T}\frac{dh_T}{d\Xi} + \frac{\delta}{\Xi} & if \quad h_T \leq h_s(\Xi), \\ \frac{\gamma+\delta}{h_T}\frac{dh_T}{d\Xi} & if \quad h_T + dh_T > h_s(\Xi + d\Xi) \\ < 0 & otherwise \end{cases} \tag{38}$$

**As it is true for $t = 1$, the property holds for any $t > 0$.(NN) implies that $\frac{1}{h_T}\frac{dh_T}{d\Xi} < \frac{1}{h_s}\frac{dh_s}{d\Xi}$ for any $T$, any $h_0$ and any sequence $(\theta_i)_{i=1}^T$. Therefore, $\frac{d}{d\Xi}G_T(h_s) > 0$ for any $T$ and $\lim_{T\to\infty}\frac{d}{d\Xi}G_T(h_s) = \frac{d}{d\Xi}\mathcal{G}(\Xi,\tau)(h_s) > 0$. This proves that $\frac{d}{d\Xi}N(G,\tau) > 0$. Moreover summing the relation (NN) on $X$ gives $dE(H) = d\left(\int hdG_T\right) < \frac{d\Xi}{\Xi}\left(\int hdG_T\right) = E[H]\frac{d\Xi}{\Xi}$. This leads to $\frac{d}{d\Xi}\left(\frac{E[h]}{\Xi N(G,\tau)}\right) < \frac{d}{d\Xi}\left(\frac{E[h]}{\Xi}\right) \Leftrightarrow \frac{d}{d\Xi}\left(\frac{E[h]}{\Xi}\right) < 0$. The function $\frac{E[h]}{\Xi N(G,\tau)}(\Xi)$ is strictly decreasing. When $\Xi$ tends toward 0, all agents choose the private sys-**

**tem. The stationary distribution is log-normal and independant of $\Xi$. As $N(G,\tau)$ tends toward zero too, the function tends toward the infinity. Conversely, when $\Xi$ is large enough, all agents prefer the public system. The stationary distribution tends toward a log-normal one and $E(h) \propto \Xi^{\delta/(1-\gamma)} \Rightarrow \frac{E[h]}{\Xi N(G,\tau)} \to 0$. Eventually, the function $\frac{E[h]}{\Xi N(\bullet)} - 1$ is strictly decreasing, continuous, because both $N(\bullet)$ and $E[h]$ (as an integrated function) are continuous. It takes both positive and negative values. There is therefore a unique value of $\Xi$ such that this function is null.**