“Metropolization” and development*

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Abstract

Using an original microeconomic survey on the demand for education in Peru, it is argued that the willingness to pay for a better quality of education is higher in urban areas and increases sharply with education attainment. A theoretical model of human capital accumulation based on these assumptions supports the existence of a development trap in Peru. This model shows that decentralizing education financing allows spatial concentration of human capital through migrations to cities, leading to development the overall country. This result is consistent with recent education trends in Peru. Eventually, the model indicates that fiscal transfers between Lima, the metropolis and other areas, could speed up the transition toward development and reduce the urban-rural gap.

Key words: human capital, development, political economy, urbanization

JEL Classification: I20, O15, O18

1 Introduction

Since Bairoch (1988), relationships between economic development and urbanization have been widely investigated. Cities appear to be the heart of social and technological evolution. Correlations between urbanization and development have been usually considered regarding output productivity and structural change. In this paper, we rather look at development as the transition toward a high quality of education.

Since the movement of independence, many developing countries have experienced difficulties to increase attainment and school quality. Widening an educational system requires indeed large resources, both financial and human and this problem may be worsened by demographic changes.[1] One of the issues emerging countries have to face is how to distribute investment in education between urban and rural areas. International evidence suggests that supply (e.g. measured by

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[1] As pointed by the world bank report (1986).

pupil-teacher ratios), demand (e.g. measured by enrollment rate) and education returns are higher in urban areas (Orazem and King, 2007). According to De la Croix and Boucekkine (2006) or Schultz (1988), urbanization favors human capital accumulation through decreasing fertility, cheaper infrastructure, lower transportation costs (owing to higher density) and higher education returns (thanks to education externalities). According to the economic literature, such externalities may play a role within schools (thanks to the so-called "peer" effect, Benabou 1996a, 1996b) or on the labor market (Acemoglu, 1996). Therefore, should governments aiming to foster development concentrate their financial resources on the main cities or subsidy education in the lower density areas?[2]

One tries to address this question in the particular case of Peru by considering a third kind of externality, the fiscal one, as in Fernandez and Rogerson (1996), Epple and Romano (1996) or Desdoigt and Moizeau (2005). This paper presents some new empirical evidence from a microeconomic survey on education demand in Peru and suggests an endogenous mechanism to explain the takeoff of education in this country.

1.1 Empirical findings on education supply and demand in Peru

There is a large consensus in the Peruvian civil society on the necessity to decentralize public investment in education.[3] Like in many other developing countries, resources devoted to education in Peru are spread unevenly between urban areas and main cities. Considering the pupil-teacher ratio in public schools as a proxy of public investment in education, recent data[4] shows indeed that the more urbanized is a region, the lower is the pupil-teacher ratio (see fig. 1 for pupil-teacher ratio in primary and secondary schools).

Figure 1: Public education supply in the regions of Peru (INEI, 2005).

Looking at human ressources, it seems that public supply of education in Peru is larger in urban areas than in rural ones. Although the Peruvian civil society complains about this, the urban-rural gap in public ressources could also be induced by different education demands. Recent INEI data on school enrollment indicate that whereas attendance in primary schools is high in every regions of Peru, young people in rural areas still tend to enroll less in secondary education (see fig. 2 for enrollment ratio in secondary schools). This may be an indicator of smaller preference for education in rural areas.

Figure 2: Enrollment in secondary education in the regions of Peru (INEI, 2005)

Given the discrepancies in both demand and supply, is not surprising that education outputs, such as average schooling or literacy appear to be lower in rural areas.

Figure 3: Illiteracy and average schooling in the regions of Peru (INEI, 2005).

[2]A very useful survey of educational issues is available in the encyclopaedia of higher education, Altbach, 1991 or Clarck and Neave, 1992.

[3]The recent seminar (2009) between the World Bank and the regional government of Junin acknowledges the importance of connections between decentralizing and quality of education.

[4]INEI 2005 National census.

Area	# households	per hh. income†	per worker income†	Schooling (years)	Gender (female)	Age (years)	Quecha spoken
Rural	451	448	267	8.4	52.7%	33.6	69.2%
Urban	1268	1139	586	11	51.5%	33.4	21.0%

†Income are in current soles

Table 1: Main statistics of the EVEP survey

To test this assumption, one introduces the results of an original survey about education demand in Peru, EVEP 2006-2007,[5] based on the last available household survey sponsored by the World Bank in Peru at this date.[6] The aim of this survey was to characterize the demand for education in Peru and to calibrate a human capital growth model. It was conducted on 1719 individuals in two waves, in the provinces of Lima and Puno in 2006 and in the provinces of Cuzco and Junin in 2007 by the same enquirers, using the same methodology and questionnaire.[7]

The survey exhibits five indicators aiming to characterize the demand for education. The first two variables intended to measure the priority of educational expenditure. In a first question, the contestants had to decide how the government should split the revenue of an additional monthly tax of 100 soles per household. The indicator "PUBLIC" is the share of this additional public expenditure which should be dedicated to public education. In a second one, people were asked how they would spent an additional income they earned by chance, of 100 soles monthly. The indicator "MARGINAL" is the share of this income dedicated to private educational expenditure. Third, people should report the amount of money they really spent for their children's education. The indicator "ACTUAL" is the logarithm of the reported monthly educational expenditure for each children of the interviewed person.[8] Eventually, people should declare the amount they would spend per child if they could benefit from a long-term loan to pay for their children's education. The fourth indicator "IDEAL" is the logarithm of the ideal monthly amount of money people would allocate to each of their children's education. Finally, one can use the fact that children of the interviewed people attend or attended a private school as another proxy for education demand.

Table 2 reports OLS regressions of the first four indicators on education and control variables[9] and a probit model on the attendance in private schools.[10]

These results indicate that whatever the indicator and the specification, the propensity of

[5]The Encuesta sobre la Vision de la Educacion en el Peru has been conducted by the author and several professional enquirers during the years 2006 and 2007 thanks to the collaboration of the university Antonio Ruiz de Montoya de Lima.

[6]ENNIV, 1994.

[7]Since questions were about education and personal characteristics such as composition of family or conditions of the childhood, the delay between the two waves does not cause problems of data comparability. The only variable which could be corrected is the declared income. But inflation was controlled in 2006-2007 in Peru and moreover given answers on income are rather approximative. However in regressions, a dummy variable is introduced in order to control for unobservable differences between the two waves.

[8]This indicator is only defined for people having children. Different children of the same contestant provide several distincts observations.

[9]i.e. years of schooling, a dummy variable indicating if the person attended university, the logarithm of the declared monthly monetary income, age, area of residence, discrete indicators from 1 to 5 accounting for how much the contestant liked school, how she considers the quality of public education and how she considers the school she attended, dummies variables indicating if Quechua is her mother tongue, if she is currently studying and if her main occupation is agriculture, her number of children and eventually a variable controlling for unobservable discrepancies between the two waves of the survey.

[10]The probit and ACTUAL regressions are based on the attendance of the children of the contestant.

Dep. var.	PUBLIC	MARGINAL	ACTUAL	IDEAL	PRIVATE
Method	OLS	OLS	OLS	OLS	probit
Years of Schooling	0.56 (1.9)	−0.13 (0.5)	0.00 (0.4)	−0.01 (1.0)	– *ns*
College education	12.86** (4.1)	9.50** (3.5)	0.58** (9.9)	0.68** (7.3)	0.79** (6.9)
ln(Income)	1.95 (1.6)	1.39 (1.3)	0.08** (4.7)	0.23** (6.3)	0.51** (9.0)
Age	−0.16 (−1.6)	−0.24** (2.8)	0.00 (1.0)	−0.01 (2.0)	0.00 (0.7)
Lima	5.64 (1.3)	0.13 (0.0)	−0.03 (0.6)	0.60** (4.6)	0.21 (1.6)
Other town	1.34 (0.4)	7.80* (2.4)	−0.03 (0.5)	0.24* (2.3)	0.11 (0.7)
Liked school	2.84 (1.7)	0.60 (0.4)	−0.01 (0.5)	0.10* (2.0)	–
Quality public education	−3.72** (−3.0)	−0.24 (0.2)	−0.06** (3.4)	−0.04 (1.1)	−0.11* (2.3)
Quality own school	4.16** (2.8)	−0.48 (0.4)	0.00 (0.4)	0.01 (0.1)	–
Quechua spoken	−2.37 (−0.9)	6.21** (2.8)	−0.05 (1.5)	−0.05 (0.7)	0.13 (1.3)
Currently studying	3.33 (1.2)	6.83** (2.9)	−0.10 (1.7)	−0.08 (1.0)	–
# Children	−0.26 (−0.3)	0.76 (1.1)	−0.02* (2.3)	−0.04 (1.5)	–
Year of interview	−21.54** (−6.3)	0.03 (0.0)	−0.05 (1.2)	0.60** (5.6)	−0.14** (4.3)
Intercept	21.41 (1.8)	10.41 (1.0)	1.75** (10.7)	1.82** (5.1)	−3.84** (9.3)
N	1338	1344	1250	1177	1628
Adj. R^2	0.11	0.06	0.16	0.33	0.23

*,** indicate respectively significance at the 5% and 0.1% level

Table 2: Impact of education on several indicators of education demand

parents to invest in education increases with their level of education. Moreover, there seems to be a threshold in the preference for education at the college level. Having attended college increases indeed every indicators, whatever the duration of university studies. Regarding the urban/rural gap, it is to be noted that inhabitants of rural areas tend to declare more private education as a priority. Nevertheless, they keep investing less and would not raise their financial contribution if benefiting from a long-term loan. This gap illustrates the distance from wishes to acts of the rural population. This could be a partial explanation of its difficulty to maintain a political pressure on the central government to better the quality of education.

A propensity to invest in education which increases with education attainment can be seen as a particular case of a marginal propensity to invest, which increases with income. This very old hypothesis, Fischer (1930), Kaldor (1955) has been recently verified on US data by Lawrence (1991). Many contemporary contributions make this assumption, see Becker and Mulligan (1997), Samwick (1998) or Atkenson (1997).

1.2 Rural exodus and development

If less educated people have a lower propensity to invest in education, urban citizens, which are more educated, should support higher taxes and investment in education than rural populations.

In this paper, it is argued that the concentration of financial resources devoted to education in the main cities of Peru is induced by the political process. If people in the remote areas appear to support more public education, they do not seem able or willing to finance it. Therefore, the education system works as if there were two different fiscal communities, the Metropolis and the Province, partially independent and financing public education. The central government invests more in areas where people pay higher taxes and have consequently more bargaining power to demand improvement of the quality of education.

Because the preference for education is heterogeneous at the national level, allowing some financial independence to highly urbanized regions can favor their convergence to development. From this point of view, education development in Peru arises from the progressive spatial extension of the main city, Lima, by attracting migrants from rural provinces and concentrating investments. One denotes this process "metropolization" .

Section two introduces the theoretical model. The third one details the condition of convergence toward development when preferences for education are increasing with human capital. Section four explains this metropolization process and section five considers the effects of fiscal transfers between the Metropolis and the rest of the country.

2 The model

2.1 Main assumptions of the framework

The following model which aims to describe and better understand the recent dynamic of human capital in Peru relies on four major assumptions.

(1) There are two sectors in the economy. The educational one and the production one which output is a final consumption good. The production in the final consumption sector Y uses only aggregated human capital H. Total factor productivity is denoted A.

$$Y(H) = AH \tag{1}$$

Maximization of profit in the competitive labor market of the non educational sector sets the wage per unit of human capital $w = A$. When the labor market is at the equilibrium, people are indifferent to the kind of occupation they will have and the income y^i of the agent i only depends of his human capital endowment h^i:[11]

$$y^i = wh^i \tag{2}$$

(2) Endowment in human capital depends of parental human capital, of the educational expenditure E and the average human capital H in the economy. Human capital law of motion within a dynasty i at the generation t has therefore the following form:[12]

$$h^i_{t+1} = \phi\left(h^i_t, E^i_t, H_t\right) \tag{3}$$

The data presented in the first section allow to calibrate the function ϕ for Peru.

(3) The level of public expenditure in education relies on a political equilibrium: first public expenditure is directly financed through a flat tax on income and second the tax rate is

[11] This view is consistent with the human capital theory and is supported by empirical evidence presented in the next section.

[12] A justification of this theoretical form is provided further.

determined by a majority vote. Consequently the expenditure depends of the aggregate income Y_t and the distribution of human capital in the economy $G_t(h)$.

$$E_t^i = \tau Y_t = P(G_t) \tag{4}$$

(4) Eventually the voter behavior reflects his own preference for education: his vote derives from the maximization of his utility. The utility function is assumed to vary with human capital h_t, the kind of area the voter lives in, his current consumption c_t and the future income of his heir y_{t+1}. Therefore, the preferred tax rate of the voter τ_t^i is:

$$\tau_t^i = \arg\max U(c_t, y_{t+1}) \tag{5}$$

The utility function is supposed to be separable in the present consumption of the household and the future consumption of the heir.

$$U(c_t, y_{t+1}) = u(c_t) + \beta(h_t)E\left[u\left(y_{t+1}\right)\right] \tag{6}$$

Households preferences are allowed to be heterogeneous and defined by an altruistic parameter $\beta(h)$.[13]

These assumptions although simplistic are classic in the political choice theory. Here is considered a continuum of heterogeneous agents who differ according to their endowment in human capital h and ability θ. The parameter θ is stochastic and represents the pure ability of the child. It is assumed independent of parental background and expenditure and follows a log-normal law. It is assumed without loss of generality that its expectancy equals $e^{\frac{\omega^2}{2}}$ and the standard deviation of $\ln\theta$ is denoted ω.

This model is then close to Glomm and Ravikumar's (1992) although preference for education, measured by the parameter β is not homogeneous among the population. Perotti (1993), Epple and Romano (1996), Fernandez and Rogerson (1996) or Desdoigt and Moizeau (2005) treat similar situations in political economy models where preferences are heterogeneous. If social classes are defined as groups of individuals sharing similar political interests, using a continuum of human capital endowments allow to define social classes whose size are endogenous. With this feature social classes are not presupposed but endogenously determined in the present analysis contrary to the previously quoted studies. This allows to track precisely the dynamic of human capital in the economy and to identify potential factors of underdevelopment trap.

In this model, the impact of endogenous fertility on human capital accumulation is not considered.[14] The population growth rate is assumed to be null for all households.[15]

Each agent lives two periods. An agent born in $t-1$ belongs to the generation t. In the first period, he goes to school a time ν and works a time $(1-\nu)$ in the firm of its parent. His productivity is supposed to be smaller than the productivity of an adult and depends of his parent's human capital h, so that he brings an additional income y_c to the household, with $c_1 < 1$:

$$y_c = c_1(1-\nu)wh \tag{7}$$

[13] A joy of giving altruism is considered here. In a more general frame, the agent should consider the utility of his child. But this specification would lead to an infinite horizon maximization problem. This complicates the calculation without modifying the properties of the model, as long as the agent does not consider the preferences of his heirs as endogenous.

[14] According to the data, the impact of urbanization on fertility is mostly explained by education level. Introducing endogenous fertility will complicates the model without modifying its properties.

[15] This simplification does not influence the properties of the model, which hold as long as fertility remains constant among households.

At the beginning of the second period, at t, he gives birth to a single child. Then, he votes for a tax rate τ in order to finance public education. After that, he works and gets an income y_t related to its human capital endowment. He has also to contribute to the infrastructure, transport and others indirect costs linked to his child's education by paying an exogenous fee c_0 which is proportional to its income.[16] The second period budget constraint becomes:

$$c_t = wh_t + wc_1(1-\nu)h_t - c_0 y_t - \tau y_t \tag{8}$$

The distribution of human capital in the generation t, denoted by the cumulative density $G_t(h)$ completely defines the system during the period from t to $t+1$.

In addition to that, two assumptions are made on the utility function:

(i) u is C^∞, $u' > 0$ and $u'' < 0$.

(ii) β is C^∞, $\beta' > 0$ and $\exists M > 0$ such that $|\beta| < M$ for all individuals.

2.2 Building the human capital accumulation process

In Peru, private education represents a significant part of the educational supply[17] but most of private schools are concentrated in Lima and in other urban areas.[18] The model considers nevertheless a pure public system, where educational expenditure are collected through a tax and distributed equally among the population. This description of the educational system is relevant if it assumed that the private education sector would shrink if the quality of public education were better. Two empirical findings support this hypothesis. First, EVEP data reveal that private schools are chosen by parents partly because of the bad quality of public education. The quality perceived have indeed a negative impact on private schooling and private expenditure (see table 2). Second, the quality of private education, measured by education returns differs only from public education due to a higher level of expenditure. There are no specific positive effect of the private education sector on the education returns according to EVEP data (see table 3).

There are mainly two types of education expenditure, infrastructure and compensations of teachers. Costs for building and managing infrastructures could be viewed as fixed costs because they do not vary with school time. These costs may be lowered by technological progress and their marginal productivity tends toward zero when they are sufficiently high to guarantee a decent quality of infrastructures. Therefore the effects of infrastructure costs will be neglected.[19]

It may possible to better education quality by lengthening the duration of schooling ν, reducing the size of class (through lower pupil-teacher ratios l) or improving the qualification of teachers (h_T). Parental human capital h_P and peer children endowment (h_C) (Benabou, 1996) are also very important educational inputs. In a general framework one may therefore choose the following form for human capital accumulation where Q represents the educational technology.[20]

$$h = Q\theta\nu^\zeta h_P^\gamma h_T^\delta h_C^\chi l_t^{-\phi} \tag{9}$$

[16]Transport, uniform and books expenditure can easily be assumed as proportional to income. Infrastructure is assumed to be financed through an exogenous flat tax.

[17]about 10.25% according to the EVEP survey.

[18]According to EVEP data, 15% of interviewed people declared having studied in private schools in Lima, 6.5% in other urban areas and 4.2% in rural areas.

[19]It is true that in developing countries like Peru, the quality of school infrastructures may be insufficient especially in rural area. But infrastructures productivity is heavily increasing with others inputs so that insufficient infrastructure is rather a consequence of a lack of human capital in the educational system than the opposite.

[20]Educational technology may have increased with discoveries easing knowledge accumulation such as paper, blackboard, printing and computer science for instance.

Because of high spatial socioeconomic stratification in Peru, it is likely that all children of a same school share the same social background. Assuming that $h_C = h$ it leads to the condition $\chi = 0$. It is obvious that compensations of teachers are proportional to school time. Moreover, in a general equilibrium framework, teachers are supposed to get paid regarding their marginal productivity and their compensation should be proportional to their human capital. These educational expenditure are also inversely proportional to the size of school classes. The marginal educational expenditure by child is then:

$$E = \frac{\nu w h_T}{l} \tag{10}$$

Substituting h_T from (10) in (9) leads to:

$$h = Q'\theta\nu^{\zeta}l^{-\phi}h_P^{\gamma}E_t^{\delta} \tag{11}$$

This equation leads to the Glomm and Ravikumar's specification in the case where $\phi = 0$, which means that education quality only depends of teacher's human capital by pupil. With this assumption the human capital accumulation process becomes:

$$h = Q\theta h_P^{\gamma}\nu^{\zeta}\left(\frac{\nu h_T}{l_t}\right)^{\delta} \tag{12}$$

With this formula, human capital productivity is independent of labor productivity.[21] It is assumed here that $\gamma + \delta < 1$, which seems consistent with estimates on EVEP data (see table 3) and with the fact that usually human capital shows large diminishing returns among time.[22]. When the educational budget is balanced, fiscal revenue equals expenditure:

$$\tau H = \frac{\nu h_T}{l} \tag{13}$$

Plugging (13) into (12), the accumulation equation becomes for a dynasty i at the generation t:

$$h_{t+1}^i = Q\theta_t^i h_t^{i\gamma}\nu_t^{i\zeta}\left(\tau_t H_t\right)^{\delta} \tag{14}$$

Moreover, maximization of education quality under the second period budget constraint gives the optimal time of schooling ν. Is is assumed here that the time of schooling does not differ between pupils and is therefore determined by the management of the academic system.[23]

$$\nu = \min\left(\frac{\zeta}{\delta}\frac{\tau}{c_1}, 1\right) \tag{15}$$

Indeed, higher opportunity cost c_1 to go to school in rural areas is a traditional explanation of lower enrollment rates. Using (15), the school time ν can be replaced in the accumulation equation (14) where the scale parameter Q differ whether the agent lives in rural area ($r = 1$) or not ($r = 0$). The accumulation function now depends only of the level of tax τ:

$$h_{t+1}^i = Q\left(r_t^i\right)\theta_t^i h_t^{i\gamma}\tau_t^{\delta+\zeta}H_t^{\delta} \tag{16}$$

[21] This is only true in the long-run because in the short-run, labor productivity could lower infrastructure costs.

[22] Although some evidence supports increasing returns of education at the microeconomic level (UNESCO 2003), social returns of human capital are usually not very large in cross-country regressions (Mankiw, Romer and Weil, 1992).

[23] Some pupils may decide to drop out school, in order to start working or for other reasons. It is however assumed here that the participation is homogeneous within areas (rural or urban) and only derives from opportunity costs.

However to ease the calculations, one can assume that because of large discrepancies in personal and average human capital between rural and urban areas, differences in school time may be linked to differences in local endowment.[24]

$$Q\left(r_t^i\right) = Q\left(r_t^i\right) h_t^{i\nu} H_t^{\psi} \tag{17}$$

One obtains then the human capital intergenerational accumulation process:

$$h_{t+1}^i = Q\theta_t^i h_t^{i\gamma} \tau_t^{\delta+\zeta} H_t^{\delta} \tag{18}$$

This form can be broaden to introduce directly educational expenditure:

$$h_{t+1}^i = Q\left(r_t^i\right) \theta_t^i h_t^{i\gamma} E_t^{i\delta+\zeta} H_t^{-\zeta} \tag{19}$$

In the followings, EVEP data are used to calibrate this equation.

Dep. var. Method	Schooling OLS	Income OLS	Income OLS	Income OLS	Income OLS	Income IV
Father's schooling	0.2** (6.4)	–	–	–	–	–
Mothers's schooling	0.18** (5.5)	–	–	–	–	–
Lima	0.8** (3.1)	0.75** (13.3)	0.77** (12)	0.75** (10.5)	0.43** (4.9)	0.15 (1.1)
Other town	1.91** (6.2)	0.29** (4.4)	0.32** (4.1)	0.33** (4)	0.05 (0.5)	−0.25 (1.6)
Years of Schooling	–	0.08** (14.2)	0.06** (9.7)	0.07** (9.5)	0.05** (6.9)	0.04** (3.5)
Age	−0.03** (3.4)	0.02* (2.2)	0.03** (3.3)	0.04** (3.4)	0.02* (2.3)	0.07** (4.3)
Age^2	–	0 (1.3)	−0.0002* (2.2)	−0.0004* (2.4)	0 (1.4)	−0.0006** (3.5)
ln(#Workers)	–	0.37** (8)	0.37** (7.3)	0.29** (4.8)	0.32** (5.5)	0.36** (4.3)
$\ln E_h$	–	–	–	0.24** (4.6)	0.2** (4)	0.45**† (4)
Private school	–	–	0.64** (5.5)	–	–	–
Farmer	–	–	–	–	−0.58** (6.3)	−0.89** (4.8)
Independent	–	–	–	–	−0.36** (3.4)	−0.36** (2.6)
Teacher	–	–	–	–	0.5** (3.4)	0.34 (1.9)
Executive	–	–	–	–	0.62** (3.7)	0.32 (1.6)
Intercept	7.88** (21.1)	4.27** (20.1)	4.08** (18.1)	3.41** (12.3)	4.32** (15.3)	3.23** (7.5)
N	1322	1198	983	727	716	353
Adj. R^2	0.23	0.35	0.35	0.34	0.41	0.38

†Instruments are education levels of the parents and whether or not the mother attended private school.
*,** indicate respectively significance at the 5% and 0.1% level

Table 3: Calibration of the human capital accumulation process

Estimates of this equation are calculated in two steps. First, impact of parental education is estimated by regressing schooling on parents' schooling (see table 3). It appears that the type

[24]This simplification is not necessary to obtain the following results although it shortens the calculations.

of school, private or public, or the fees have no impact on years of schooling. This regression allows to estimate that the parameter γ is around 0.4.

To estimate the impact of educational expenditure on education returns, one introduces the logarithm of fees in a standard Mincerian regression. The expenditure is supposed to be equal to 7 soles per month in public schools.[25] However, the educational expenditure of the contestant may be endogenous especially when the contestant is not the head of household or when he actually works and contributes to the household's income. To limit that problem, the regression is also run only when the contestant or his spouse is the head of the household and instrumented by educational characteristics of the head of household's parents. The IV estimates is still strongly significant and even higher than the OLS one, although the number of observations becomes quite limited.[26] These regressions allow to determine that the parameter $\delta + \zeta$ ranges between 0.2 and 0.4. E_h stands for the educational expenditure declared to have been received by the head of household.

The parameter ζ cannot be identified by microeconomic regressions, as the evolution of macroeconomic human capital among time is not well known. In the followings this framework is used to derive the optimal behavior of the agents. The next section exhibits the conditions under which the economy can be trapped in a low equilibrium when education is financed at the national level. However, it is worth noting that the following properties do not depend of the values of the parameters.

3 A development trap

Several studies have acknowledged that education finance may be related to political economy issues.[27] In this section, one shows that under specific conditions, it would be impossible for a country like Peru to increase public investment in education simply because there is no majority among the population supporting higher taxes to better the quality of education. It that case, the development of the country would be prevented by insufficient education; it would be locked in a "trap". The following property presents sufficient assumptions guaranteeing the existence and uniqueness of the political equilibrium.

Proposition 1 *In a public system as described previously and when (i) and (ii) are verified, there is a unique political equilibrium at each time and the tax rate is defined by the nullity of the marginal utility of the median voter.*

Proof Plugging the accumulation equation (18) into (6), utility can be expressed as a function of τ only:

$$U(\tau) = u\left(wh\left(1 + (c_1 - c_0) - \tau(1 + \frac{\zeta}{\delta})\right)\right) + \beta(h)E\left[u\left(wQ\theta_t h_t^\gamma \tau_t^{\delta+\zeta} H_t^\delta\right)\right] \tag{20}$$

The marginal utility, when $\nu < 1$ and $\tau < 1$ is continuous and strictly decreasing as U''_τ is well defined and negative because of $u'' < 0$ and $u' > 0$:

$$U''_\tau = \left(wh(1 + \frac{\zeta}{\delta})\right)^2 u''(c) + \beta(h)\frac{(\delta + \zeta)}{\tau^2}\left((\delta + \zeta)E\Big[y_{+1}^2 u''(y_{+1})\Big] - (1 - \delta - \zeta)E\Big[y_{+1}u'(y_{+1})\Big]\right) \tag{21}$$

[25]This figure has been calculated to be comparable with level of expenditure in private schools at the time of the survey.

[26]This could be due to the fact that the contestant is usually older when he is the head of household (or his spouse). The elasticity of education to educational expenditure may indeed decrease as attainment increases and human capital disparities fade.

[27]See Bowman and alli., 1986 for empirical cases or Zhang, 2008 for a theoretical model.

The marginal utility always takes positive values for a sufficiently small tax rate. As $\lim_{x\to 0} u'(x) > 0$,

$$\lim_{\tau\to 0} U'_\tau = -wh(1+\frac{\zeta}{\delta})u'(c) + \beta(h)\frac{\delta+\zeta}{\tau}E\Big[y_{+1}u'(y_{+1})\Big] = \infty > 0 \qquad (22)$$

If $\lim_{\tau\to 1} U'_\tau > 0$, the marginal utility is always increasing. Otherwise, the marginal utility takes both positive and negative values and there is a unique τ for each agent which maximizes its utility. Thus, preferences are always single-peaked and the medium voter theorem can be applied. □

Although there is a single political equilibrium at each period, there can be several stationary distributions of human capital. In that case, the long-term quality of education in the country will depend of initial conditions. The following property underlines some sufficient properties utility and educational technology have to verify, so that several stationary distributions exist. Let us consider the additional conditions for the utility function:

(iii) $\rho(x) = -\frac{xu''(x)}{u'(x)} \geq 1$

(iv) $\forall\{x,y\}\in]0,\infty[, \min\left(\frac{1-\rho(x)}{1-\rho(y)}\right) \geq \gamma+\delta$

(v) $\max\Big(h\frac{\beta'}{\beta}\Big) > \frac{1-\gamma-2\delta}{\delta} + \max(\rho)\left(1+\left(\frac{1-\delta-\gamma}{\delta}\right)\frac{\tau^*}{1+(c_1-c_0)-(1+\frac{\zeta}{\delta})\tau^*}\right)$, where τ^* is a upper bound for the investment rate in education.

Proposition 2 *When preferences verify* (i) *to* (v)*, there exist* $\underline{Q}$ *and* $\overline{Q}$ *such that the system admits more than one stationary distributions if and only if* $\underline{Q} < Q < \overline{Q}$.

The proof can be found in appendix. Assumptions (iii) and (iv) are classical. The key assumption (v) seems to be verified in the Peruvian situation. Indeed the propensity to invest in children's education increases with parents' human capital and moreover presents a threshold at the university level. The following property demonstrates that education in the country always converges in the long-run to a specific stationary distribution.

Proposition 3 *When conditions* (i) *to* (v) *are verified, there are always at least two stable stationary distributions and the system always converges to one of them, whatever the initial distribution.*

The proof can be found in appendix. The next corollary gives a sufficient condition on the initial distribution so that the economy converge to the highest distribution.

Corollary 1 *When there are only two stables distributions and the initial distribution is log-normal with a median* $\mu_0 = E[\ln(h)]$ *and a standard deviation* $\sigma_0^2 = Var[\ln(h)]$*, there exists a decreasing and continuous function* $\mathcal{C}$ *such that the system converges to the high distribution if and only if* $\mu_0 > \mathcal{C}(\sigma_0)$.

The proof can be found in appendix. This property indicates that an economy which is initially not enough endowed in human capital may be locked in a development trap. In the next section, one shows that whatever the initial distribution, there always is a strategy to manage human capital so that the entire economy would converge to the highest distribution[28]. One may call this strategy "metropolization" because it is based on the progressive transfer of population from the provincial areas to the main city where human capital is concentrated.

[28] In the followings "convergence to development" will means that the human capital distribution of the economy converges to the highest stationary distribution in the long-run.

4 Convergence because of concentration

Convergence to development is determined by the initial distribution. In order to leave the trap, it may be useful to divide the economy into two independent communities by regrouping the most endowed agents in the same group. Once one of the community has converged to development, it is possible to enlarge it by transferring population from the developing community to the developed one. This process appears to describe correctly the recent development pattern of the Peruvian economy. Economic and human capital growth have been concentrated in the main city, Lima, which became bigger and bigger by attracting migrants from the rural areas.

Proposition 4 *For any initial distribution of human capital, it is possible to split the population in two communities, such that one of them at least converges to development.*

Proof It is supposed for more simplicity that before the partition the distribution of human capital is log-normal.[29] According to the previous corollary, a distribution with s. d.σ_0 will converge to the highest stationary one if and only if its median $\mu_0 > \mathcal{C}(\sigma_0)$. Let us consider the community formed with the people whose human capital is included in $[h_1, h_1 + dh]$, with $dh \approx 0$. After one period, the distribution of human capital in this community is log-normal, with a standard deviation ω and a median $\mu_1 = (\gamma + \delta)\ln(h_1)$. Choosing $h_1 > \exp\left(\frac{\mathcal{C}(\omega)}{\gamma+\delta}\right)$ is sufficient to guarantee that this new community will converge to development. □

The following proposition shows that polarizing growth of human capital in the metropolis is an efficient strategy to develop the country rapidly, although migrations to the city should be contained.

Proposition 5 *When $\underline{Q} < Q < \overline{Q}$, it is possible to lead the two communities to development by transferring at each period a share $\Delta\xi$ of population from the developing community to the developed one.*

Proof The potential stationary distributions are defined by their median. They are the zeros of the following function M, which is continuous in μ and Q and where $\mathcal{T}(\mu)$ is the Condorcet equilibrium tax rate associated with the log-normal distribution of median μ:

$$M(\mu, Q) = \mu - \frac{ln(Q) + \delta\frac{\sigma^2}{2} + \delta\mathcal{T}(\mu)}{1 - \gamma - \delta} \tag{23}$$

The condition $\underline{Q} < Q < \overline{Q}$ implies that the highest stationary distribution is stable because the highest zero of M, $\overline{\mu}$ is above μ_c, the highest development threshold, which is the highest zero of M such that $M'_\mu < 0$.

Let us consider the metropolis N, which distribution is log-normal with a standard deviation σ and a median μ^N. As N has converged to development, $\mu^N > \mu_c$. To prove that the metropolis will still converge to development after the arrival of the emigrants, it is sufficient to prove that the asymptotic median μ^N_∞ remains above μ_c. Let us denote ξ the share of the national population who lives in the metropolis at $t = 0$ and $\Delta\xi$ the population E who migrates to N at this time. Among the natives N, the distribution of human capital remains log-normal with a standard deviation σ at each time after the arrival of the emigrants. As all emigrants have a positive human capital ($h^i_t \geq 0, \forall i \in E$) at each time, a lower bound for the median voter's human

[29] If it is not the case, it has already been proved that after some time in a public system, the economy will converge to a log-normal distribution with standard deviation σ.

capital $h_t^{M,m}$, which is a native, can be linked to the median of the distribution of human capital among the natives μ_t^N:

$$ln(h_t^{M,m}) \geq \mu_t^N + \sigma\Phi^{-1}\left(\frac{1}{2} - \frac{\Delta\xi}{2\xi}\right) \tag{24}$$

Using the accumulation equation, one can also obtain a lower bound for the median human capital among the natives N at the beginning of the next period, μ_{t+1}^N:

$$\mu_{t+1}^N \geq \ln(Q) + \gamma\mu_t^N + \delta\ln\left(e^{\mu_t^N + \frac{\delta\sigma^2}{2}}\frac{\xi}{\xi + \Delta\xi}\right) + \delta\mathcal{T}\left(\mu_t^N + \sigma\Phi^{-1}\left(\frac{1}{2} - \frac{\Delta\xi}{2\xi}\right)\right) \tag{25}$$

When $\Delta\xi \ll \xi$, a Taylor development at the first order in $\frac{\Delta\xi}{\xi}$ leads to:

$$\mu_{t+1}^N \geq \ln(Q) + (\gamma + \delta)\mu_t^N + \delta\frac{\sigma^2}{2} - \delta\frac{\Delta\xi}{\xi} + \delta\mathcal{T}(\mu_t^N) - \delta\sigma\sqrt{2\pi}\mathcal{T}'(\mu^N)\frac{\Delta\xi}{2\xi} \tag{26}$$

Let us denote $Q' = Q\exp\left(-\delta\left(1 + \frac{\sigma}{2}\sqrt{2\pi}\mathcal{T}'(\mu^N)\right)\frac{\Delta\xi}{\xi}\right)$ to define $\mu^{N\prime}$ as the zero of $M(\mu, Q')$. As Q' is continuous in $\frac{\Delta\xi}{\xi}$, M is continuous in $\frac{\Delta\xi}{\xi}$. Thus $\mu^N > \mu_c$ implies that there exists a $\varepsilon > 0$, such that for all $\frac{\Delta\xi}{\xi} < \varepsilon$, $\mu^{N\prime} > \mu_c$. The property 2 indicates that the distributions of human capital within the natives N and the emigrants E converge to the same log-normal distribution, which asymptotic median μ_∞^N verifies:

$$\mu_\infty^N \geq \mu^{N\prime} > \mu_c \tag{27}$$

Using corollary 1, we know that the whole metropolis converges to development. A limited transfer of population at each period, does not modify the political equilibrium of the metropolis. Moreover, as the maximal value of $\frac{\Delta\xi}{\xi}$ is independent of ξ, the growth rate of population in the metropolis can be held constant and this strategy allows to lead the entire economy above the development trap in a finite time. □

It is also easy to show that when the size of N is sufficiently small, a sufficiently large transfer of population from the provinces will bring down N below the development threshold. Transfer of population between the two communities should be sufficiently gradual to guarantee the convergence of the overall economy to development. The higher economic growth induced by the rise of education returns contributes to attract migrants from the provincial areas, where education quality is lower.[30]

Nevertheless, the concentration of educational resources in the main city induces protests among the civil society and the rural populations, which complains about equity issues. To mitigate increasing inequalities during the transition period, one may argue in favor of fiscal support from the metropolis to the provincial areas. In the next section ones tries to determine whether this could be an efficient way to speed up convergence and to reduce inequalities.

5 Fiscal transfer between regions to speed up development

5.1 Impact of fiscal transfers on the speed of convergence

Let us consider two regions, the metropolis M and the province P, which are fiscally independent. In each region, citizens vote for the level of a flat tax on income in order to finance public education. To simplify the calculations we make the following hypothesis on the utility function:[31]

[30] Like in Todaro's model.

[31] This framework is consistent with the results of the EVEP survey.

(vi) $u(x) = \ln(x)$, the utility function is logarithmic;

(vii) $\beta(x) = \underline{\beta} + \left(\overline{\beta} - \underline{\beta}\right) \lim_{\chi \to 0} \Phi\left(\frac{\ln h - \ln h_c}{\chi}\right)$

There are only two types of preferences for the education in the economy, a large one $\overline{\beta}$ and a small one $\underline{\beta}$. This preference depends whether the agent's endowment is below or above the poverty threshold h_c. To simplify the analysis, opportunity costs are not considered although this assumption has no impact on the results. With these specifications, the previous properties are verified. Let us consider that an exogenous tax s is collected in the metropolis, to finance fiscal transfers in order to better the quality of education in the provincial areas. This tax rate is chosen by the central planner previous to the vote on the revenue financing education τ. The median voter has a large preference for education in the metropolis and a small one in the province. As a consequence, when there are no transfers, the equilibrium tax rates are:

$$\overline{\tau} = \frac{\overline{\beta}\left(\delta + \zeta\right)}{1 + \overline{\beta}\left(\delta + \zeta\right)}; \underline{\tau} = \frac{\underline{\beta}\left(\delta + \zeta\right)}{1 + \underline{\beta}\left(\delta + \zeta\right)} \tag{28}$$

The utility of the median voter of the metropolis becomes:[32]

$$u_s^m\left(\tau_M\right) = \ln\left(1 - \tau_M\right) + \overline{\beta}\left(\delta + \zeta\right)\ln\left(\tau_M\left(1 - s\right)\right) \Rightarrow \tau_M\left(s\right) = \overline{\tau} \tag{29}$$

The investment rate in the metropolis diminishes because of the transfer and becomes $\overline{\tau}\left(1 - s\right)$. Let us denote s' the subsidy received by the province as a share of its GDP. This subsidy is crowding out local financing:

$$u_s^m\left(\tau_P\right) = \ln\left(1 - \tau_P\right) + \overline{\beta}\left(\delta + \zeta\right)\ln\left(\tau_P + s'\right) \Rightarrow \tau_P\left(s\right) = \underline{\tau} - \frac{s'}{1 + \underline{\beta}\left(\delta + \zeta\right)} \tag{30}$$

The investment rate slightly increases in the Province to $\tau_P = \underline{\tau}\left(1 + s'\right)$. The level of subsidy is determined by the balance between the tax collected in the Metropolis and the subsidy received by the Province. It depends of the relative productivity in each regions and their relative size in terms of population, ξ.

$$\overline{\tau} s H_M \xi = \underline{\tau} s' H_P\left(1 - \xi\right) \tag{31}$$

The central planner can reduce the discrepancies in educational investment between the two regions using fiscal transfer. But such a transfer reduces human capital in the metropolis and then the median voter's endowment. Therefore, the higher possible rate of transfer s the central planner can chose without jeopardizing the stability of the political equilibrium in the metropolis is such than the median voter remains above the threshold h_c. As long as migrations keep the metropolis growing, it allows the government to increase preogressively their support to the province and to mitigate human capital related inequalities.

Let us consider stationary distributions of human capital in the two regions. Both distributions are log-normal, with the same standard deviation σ but different median:

$$\mu_M = \frac{q + \delta/2\sigma^2 + \left(\delta + \zeta\right)\ln(\overline{\tau}\left(1 - s\right))}{1 - \gamma - \delta} \tag{32}$$

$$\mu_P = \frac{q + \delta/2\sigma^2 + \left(\delta + \zeta\right)\ln\left(\underline{\tau} + s\overline{\tau}\frac{\xi}{1-\xi}e^{\left(\mu_M - \mu_P\right)}\right)}{1 - \gamma - \delta} \tag{33}$$

[32]Exogenous variables are omitted to ease the presentation.

The difference between those medians $\Delta = \mu_M - \mu_P$ is the zero of the implicit function $\mathcal{D}$. The problem only depends of the parameters $a \equiv \frac{(\delta+\zeta)}{1-\gamma-\delta}, \overline{\tau}, \underline{\tau}$, σ and the "poverty" threshold, h_c.

$$\mathcal{D}(\Delta, s) = \Delta - a\left(\ln\left(\overline{\tau}(1-s)\right) - \ln\left(\underline{\tau} + \overline{\tau}s\frac{\xi}{1-\xi}e^{\Delta}\right)\right) \tag{34}$$

This function has only one zero, which is decreasing with s. Obviously, human capital inequalities decrease with the level of subsidy. To minimize the speed of convergence, the central planner has to minimize the threshold value of ξ such that the share of the entire population above the poverty line h_c remains above $\frac{1}{2}$. This is equivalent to search for the value of s which maximizes the share of the population v above the poverty line for a given spatial distribution of the population ξ.

$$v(s) = \xi\Phi\left(\frac{\mu_M(s) - \ln h_c}{\sigma}\right) + (1-\xi)\Phi\left(\frac{\mu_P(s) - \ln h_c}{\sigma}\right) \tag{35}$$

Proposition 6 *For any parameters of the model $\xi, a, \overline{\tau}, \underline{\tau}, \sigma$ fiscal transfer from the metropolis to the province can speed up the transition to development as long as initially* $\ln h_c < \frac{\underline{\mu}+\overline{\mu}}{2}$.

Proof $\frac{d\mu_M}{ds} = -\frac{a}{1-s}$ comes directly. The implicit function theorem allows to calculate:

$$\frac{d\mu_P}{ds} = \frac{a\tau_M\xi/(1-\xi)e^{\mu_M-\mu_P}\left(1+s\frac{d\mu_M}{ds}\right)}{e^{\mu_P/a} + as\tau_M\xi/(1-\xi)e^{\mu_M-\mu_P}} \tag{36}$$

Using (36), the share of the overall population v supporting large educational expenditure (35) evolves like:

$$\frac{dv}{ds} = \xi a\left(-\frac{1}{1-s}\Phi'\left(\frac{\mu_M(s) - \ln h_c}{\sigma}\right) + \Phi'\left(\frac{\mu_P(s) - \ln h_c}{\sigma}\right)\frac{\tau_M e^{\mu_M-\mu_P}\left(1-\frac{s}{1-s}a\right)}{e^{\mu_P/a} + as\tau_M\frac{\xi}{1-\xi}e^{\mu_M-\mu_P}}\right) \tag{37}$$

When the subsidy tends toward zero, one can use $\lim_{s\to 0} e^{\mu_P/a} = \tau_P$ and $\Phi'\left(\frac{x}{\sigma}\right) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left\{-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2\right\}$ to calculate:

$$\lim_{s\to 0}\frac{dv}{ds} \propto -\exp\left\{-\frac{1}{2}\left(\frac{\mu_M - \ln h_c}{\sigma}\right)^2\right\} + \exp\left\{-\frac{1}{2}\left(\frac{\mu_P - \ln h_c}{\sigma}\right)^2\right\} e^{(\mu_M-\mu_P)\left(1+\frac{1}{a}\right)} \tag{38}$$

Which leads to the sought condition

$$\lim_{s\to 0}\frac{dv}{ds} > 0 \Leftrightarrow (\overline{\mu} - \underline{\mu})\left(\frac{\underline{\mu}+\overline{\mu}}{2} - \ln h_c + \left(1+\frac{1}{a}\right)\sigma^2\right) > 0 \tag{39}$$

□

When the development threshold is sufficiently low, the share of voters around the country supporting high taxes to finance high quality education can be increased by positive transfers from the metropolis. To have a more precise idea about the actual impact of transfers on the speed of convergence one should run numerical simulations.

Elasticities	Intergenerational	Expenditure	School time	School size	s.d.⋆
Parameters	γ	δ	ζ	ϕ	σ
Values	0.4	0.2	-†‡	0‡	0.55

⋆ Standard deviation of the human capital distribution in log †not identified ‡Assumed to be zero

Table 4: Calibrated value of the human capital accumulation process.

5.2 Numerical simulations

The value of $\overline{\tau}$ and $\underline{\tau}$, the preferred tax rates of the population, cannot be correctly extrapolated from the survey.[33] To calibrated those parameters, one may consider that in the more advanced countries, the investment rate in education is about 6-7% of GDP[34] on average, which gives a $\overline{\tau}$ parameter around 0.06. The investment rate in public education was 2.2% in Peru, before the burst of private education of the 90's. It is around 3.3% today. Therefore, $\underline{\tau}$ will be set to 0.02 as it represents the preferred tax rate of the poorest share of the population.

The value of the threshold level h_c is probably close to the median value of human capital of the highest distribution. Indeed, it appears in the EVEP data that the preference for education strongly increases when people have entered university. The threshold value is therefore high compared to the distribution of education levels in the country.[35] Using (35) for $s = 0$ and $v = \frac{1}{2}$ one can define the "critical" size of the metropolis ξ_0^* such that as soon as the metropolis is bigger than ξ_0^* there is a majority of voter in the country supporting a high tax rate to finance education. At this point, centralizing the fiscal system provokes the convergence to development of the entire country. ξ_0^* is therefore a measure of the time needed to complete the convergence to development. The equation (40) shows that h_c is a continuous and increasing function of ξ_0^*.

$$\xi_0^* = \frac{\frac{1}{2} - \Phi\left(\frac{\underline{\mu} - \ln h_c}{\sigma}\right)}{\Phi\left(\frac{\overline{\mu} - \ln h_c}{\sigma}\right) - \Phi\left(\frac{\underline{\mu} - \ln h_c}{\sigma}\right)} \tag{40}$$

Setting the "poverty" threshold is equivalent to set the "critical" size of the metropolis.

When transfers are not null, one defines the critical size of the metropolis ξ^*, as the minimum value of the size of metropolis which guarantees that there will be a majority of voters in the economy in favor of high taxation for education in the long-run.

Whatever the considerations on inequalities, there is an obvious limit on fiscal transfers. The net investment in the metropolis should remain sufficiently high to guarantee the stability of the political equilibrium. This leads to the condition:

$$s < 1 - \left(\frac{\overline{\tau}}{\underline{\tau}}\right)^{\xi_0^* - 1} \tag{41}$$

Using the previous calibration, one can plot the evolution of the critical size of the metropolis for different values of the development threshold.[36]

The figure 4 shows indeed that limited financial transfers from the different regions could significantly lower the critical size of the metropolis. According to simulations, transfers about 10% of the collected tax for education in the metropolis would decrease the minimal size of the metropolis (as the share of the total population) of 10 percentage points.[37]

[33] To measure them precisely, one should use the results of experiments where people could choose between several services of education, whose quality and price are different.
[34] According to UNESCO data.
[35] According to the EVEP survey, only 20% of the population above 15 have reached university in Peru in 2007.
[36] defined by ξ_0^*.
[37] This results holds for different values of the parameters, as long as they do not change dramatically.

Figure 4: Evolution of the critical size of the metropolis with the subsidy from the Metropolis to the Province for different initial size of the main city

6 Conclusion

New empirical findings on the education demand and supply in Peru indicate that the propensity to invest in education is an increasing function of human capital. There seems to be especially a threshold, at the level of college education, in the propensity to pay for a better education. With such a threshold in the preferences, a country initially not enough endowed in human capital can be trapped in under development.

A theoretical model shows that decentralizing education financing can allow certain regions, such as main cities where human capital is concentrated, to escape the development trap. The progressive spatial extension of the metropolis can induce the convergence of the entire country toward a system of high quality education. This process may correspond to the current growth of the metropolis of Lima, which concentrates economic development by attracting migrants from provincial and rural areas.

In the case of Peru, numerical simulations show that limited financial transfers from the metropolis to the other part of the country could significantly fasten the transition to development. These results are consistent with the current initiatives of Peruvian authorities to decentralize financing and management of education at a regional level.

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Appendix

Proof of proposition 2

Let us consider a stationary distribution $G(h)$ of the system. Starting from this distribution, the mean human capital H and the tax rate τ are stationary. In this case, the accumulation process (18) becomes Markovian and the distribution of human capital converges to a log-normal one. Thus every stationary distributions are log-normal. Given stationary values H and τ, it is possible to determine the parameters μ and σ which characterize the stationary distribution G.

$$\begin{cases} \mu = E[\ln(h)] = \frac{\ln(Q)+\delta\ln(H)+(\delta+\zeta)\ln(\tau)}{1-\gamma-\delta} \\ \sigma^2 = Var[ln(h)] = \frac{\omega^2}{1-\gamma^2} \end{cases} \tag{42}$$

The tax rate is given by the nullity of the marginal utility (defined by (22)) of the median voter, whose human capital is h^m. It is therefore a function $\mathcal{T}(w, H, h^m)$ of h^m, the wage w and the average human capital H. This function is defined as the zero of the implicit function $\mathcal{U}(\tau, w, H, h^m)$:

$$\tau = \mathcal{T}(w, H, h^m) \Leftrightarrow \mathcal{U}(\tau, w, H, h^m) = U'_\tau(w, H, h^m) = 0 \tag{43}$$

It goes directly from (21) that $\frac{\partial \mathcal{U}}{\partial \tau} = U''_\tau < 0$. Introducing $\rho(x) = -\frac{xu''(x)}{u'(x)}$, the inter-temporal elasticity of substitution, allows to simplify the expression of $\frac{\partial \mathcal{U}}{\partial h}$:

$$\frac{\partial \mathcal{U}}{\partial h} = -w\left(1+\frac{\zeta}{\delta}\right)u'(c)\Big(1-\rho(c)\Big) + \beta(h)\frac{\delta+\zeta}{\tau}E\left[y_{+1}u'(y_{+1})\left(\frac{\gamma}{h}(1-\rho(y_{+1}))\right)+\frac{\beta'}{\beta}\right] \tag{44}$$

Using $\beta' \geq 0$, it exists y^* such that:

$$\frac{\partial \mathcal{U}}{\partial h} \geq -w\left(1+\frac{\zeta}{\delta}\right)u'(c)(1-\rho(c)) + \beta(h)\frac{\delta+\zeta}{\tau}(1-\rho(y^*))\frac{\gamma}{h}E\left[y_{+1}u'(y_{+1})\right] \tag{45}$$

Using (22) the definition of optimal τ:

$$\frac{\partial \mathcal{U}}{\partial h} \geq w\left(1+\frac{\zeta}{\delta}\right)u'(c)\Big(-\Big(1-\rho(c)\Big)+\gamma\Big(1-\rho(y^*)\Big)\Big) \geq 0 \Leftrightarrow \frac{1-\rho(c)}{1-\rho(y^*)} \geq \gamma \tag{46}$$

The condition (iv) ensures that $\frac{\partial \mathcal{U}}{\partial h} \geq 0$. Using the implicit function theorem, one obtains that $\mathcal{T}'_h > 0$. Thus the median voter is the voter which has the median human capital. As the stationary distribution is log-normal, h_m and H can be linked to the median μ.

$$\tau = \mathcal{T}(h_m, H) = \mathcal{T}(e^\mu, e^{\mu+\frac{\sigma^2}{2}}) \equiv exp\{\mathcal{T}(\mu)\} \tag{47}$$

Let us calculate $\mathcal{T}'_H$ using $\frac{\partial \mathcal{U}}{\partial H}$.

$$\frac{\partial \mathcal{U}}{\partial H} = \beta(h)\frac{\delta+\zeta}{\tau}\frac{\gamma}{H}E\left[y_{+1}u'(y_{+1})(1-\rho(y_{+1}))\right] \leq 0 \tag{48}$$

Replacing h by h_m and H by $h_m e^{\frac{\sigma^2}{2}}$ and using $\beta' \geq 0$ and the definition of τ, one obtains:

$$\frac{d\mathcal{U}}{dh_m} = \frac{\partial \mathcal{U}}{\partial H} + e^{\frac{\sigma^2}{2}}\frac{\partial \mathcal{U}}{\partial H} \geq w\left(1+\frac{\zeta}{\delta}\right)u'(c)\Big(-\Big(1-\rho(c)\Big)+(\gamma+\delta)\Big(1-\rho(y^*)\Big)\Big) \geq 0 \Leftrightarrow \frac{1-\rho(c)}{1-\rho(y^*)} \geq \gamma+\delta \tag{49}$$

The condition (iv) guarantees that $\mathcal{T}(\mu)$ is increasing. Thus the stationarity condition of the distribution can be rewritten as the nullity of the implicit function M of μ:

$$M(\mu) = \mu - \frac{q + \delta\frac{\sigma^2}{2} + (\delta+\zeta)\mathcal{T}(\mu)}{1-\gamma-\delta} = 0 \tag{50}$$

The function $\mathcal{T}$ is C^∞, bounded and strictly increasing.

$$\lim_{\mu\to-\infty} \mathcal{T}' = \lim_{\mu\to\infty} \mathcal{T}' = 0 \Rightarrow \lim_{\mu\to-\infty} M'_\mu = \lim_{\mu\to-\infty} M'_\mu = 1 \tag{51}$$

The function M has the same inflexion points as the function $\mathcal{T}$. As $\mathcal{T}$ is bounded, it is necessary convex in $-\infty$ and concave in ∞. Thus $\mathcal{T}$ and M admit an odd number of inflexion points. The function M is convex before h_i and concave after. It reaches its maximum in h_i. Then the function M is strictly increasing around h_i if and only if $\mathcal{T}'(h_i) < \frac{1-\gamma-\delta}{\delta+\zeta}$.

$$\frac{\partial\mathcal{U}}{\partial h_m} = w\left(1+\frac{\zeta}{\delta}\right)u'(c)\Big(-(1-\rho(c)) + (\gamma+\delta)(1-\rho(y^*)) + h_m\frac{\beta'}{\beta}\Big) \tag{52}$$

$$\frac{\partial\mathcal{U}}{\partial\tau} = \left(wh_m(1+\frac{\zeta}{\delta})\right)^2 u''(c) + \beta(h_m)\frac{\delta+\zeta}{\tau^2}E\left[y_{+1}u'(y_{+1})\right]\Big(-1 + (\delta+\zeta) - (\delta+\zeta)\rho(y^*)\Big) \tag{53}$$

Using the definition of τ at the equilibrium, it leads to:

$$\frac{\partial\mathcal{U}}{\partial\tau} = \frac{h_m}{\tau}w\left(1+\frac{\zeta}{\delta}\right)u'(c)\left(-\frac{\rho(c)(1+\frac{\zeta}{\delta})\tau}{1+(c_1-c_0)-(1+\frac{\zeta}{\delta})\tau} - 1 + (\delta+\zeta) - (\delta+\zeta)\rho(y^*)\right) \tag{54}$$

Using the implicit function theorem, one can explicit $\mathcal{T}'(\mu) = \frac{d\ln\tau}{d\ln h}$ for a given h such that $\mu = \ln(h)$.

$$\mathcal{T}'(\mu) = \frac{h}{\tau}\frac{d\tau}{dh} = -\frac{h}{\tau}\frac{\frac{\partial\mathcal{U}}{\partial h}}{\frac{\partial\mathcal{U}}{\partial\tau}} = \frac{-(1-\rho(c)) + (\gamma+\delta)(1-\rho(y^*)) + h\frac{\beta'}{\beta}}{\frac{\rho(c)(1+\frac{\zeta}{\delta})\tau}{1+(c_1-c_0)-(1+\frac{\zeta}{\delta})\tau} + 1 - (\delta+\zeta) + (\delta+\zeta)\rho(y^*)} \tag{55}$$

Eventually,

$$\mathcal{T}'(\mu) > \frac{1-\delta-\gamma}{\zeta+\delta} \Leftrightarrow \frac{-(1-\rho(c)) + (\gamma+\delta)(1-\rho(y^*)) + h\frac{\beta'}{\beta}}{\frac{\rho(c)(1+\frac{\zeta}{\delta})\tau}{1+(c_1-c_0)-(1+\frac{\zeta}{\delta})\tau} + 1 - (\delta+\zeta) + (\delta+\zeta)\rho(y^*)} > \frac{1-\delta-\gamma}{\zeta+\delta} \tag{56}$$

As τ is the tax rate to finance education, it can be bounded by the maximal investment rate in education around the world, which is $\tau^* = 0.13$ according to UNESCO data.

Therefore, a sufficient condition is:

$$h\frac{\beta'}{\beta} > \frac{1-\gamma-2\delta}{\delta} + \max(\rho)\left(1 + \left(\frac{1-\delta-\gamma}{\delta}\right)\frac{\tau^*}{1+(c_1-c_0)-(1+\frac{\zeta}{\delta})\tau^*}\right) \tag{57}$$

Consequently, according to (v) M' admits two zeros around μ_i, $\mu_1$and μ_2 and there exist $\mu_\bullet$and $\mu^\bullet$ such that the function M is increasing on $]\mu_\bullet, \mu_1]\bigcap[\mu_2, \mu^\bullet[$and decreasing on $[\mu_1, \mu_2]$. M admits more than one zero if and only if:

$$\begin{cases} \mu_1 - \ln(Q) - \delta\frac{\sigma^2}{2} - (\delta+\zeta)\mathcal{T}(\mu_1, Q) > 0 \\ \mu_2 - \ln(Q) - \delta\frac{\sigma^2}{2} - (\delta+\zeta)\mathcal{T}(\mu_2, Q) < 0 \end{cases} \tag{58}$$

Let us now prove that $M'_Q > 0$. It is equivalent to prove that $\frac{Q}{\tau}\frac{d\tau}{dQ} > -\frac{1}{\delta}$.

$$\frac{\partial \mathcal{U}}{\partial Q} = \beta(h_i)\frac{\delta+\zeta}{\tau}\frac{1}{Q}E\left[y_{+1}u'(y_{+1})\right](1-\rho(y^{**})) = \frac{1}{Q}wh_i(1+\frac{\zeta}{\delta})u'(c)(1-\rho(y^{**})) \tag{59}$$

$$\frac{Q}{\tau}\frac{d\tau}{dQ} = \frac{1-\rho(y^{**})}{\rho(c)+1-(\delta+\zeta)+(\delta+\zeta)\rho(y^{*})} \tag{60}$$

$$\frac{Q}{\tau}\frac{d\tau}{dQ} > \frac{-1}{\delta} \Leftrightarrow \delta(\rho(y^{**})-\rho(y^{*})) - \zeta(\rho(y^{*}-1) < \alpha\rho(c)+1 \tag{61}$$

A sufficient condition is then:

$$\forall\{x,y\}\ ,\ \max|\rho(x)-\rho(y)| < \frac{1}{\delta} \tag{62}$$

Proof of proposition 3

It is sufficient to prove that the average human capital H and the tax rate τ converge in order to demonstrate the convergence for every initial distribution. Let us note $q = \ln(Q)$, $\varepsilon_i = \ln(\theta_i)$ et $\lambda_i = q + (\delta+\zeta)\ln\tau_i + \delta\ln(H_i)$. Using a sequence, the following property can be proved for any real h_0:

$$\ln(h_{t+1}) = \gamma^{t+1}\ln(h_0) + \sum_{i=0}^{t}\gamma^{t-i}\lambda_i + \sum_{i=0}^{t}\gamma^{t-i}\varepsilon_i \tag{63}$$

Using the definition of the process (18), the previous property is verified at the first rank. Let us suppose that it holds for any $t \leq s$. Therefore,

$$\ln(h_{s+1}) = \gamma\ln(h_s) + q + (\delta+\zeta)\ln(\tau_s) + \delta\ln(H_s) + \varepsilon_s \tag{64}$$

$$= \gamma(\gamma^s\ln(h_0) + \sum_{i=0}^{s-1}\gamma^{s-1-i}\lambda_i + \sum_{i=0}^{s-1}\gamma^{s-1-i}\varepsilon_i) + \lambda_s + \varepsilon_s = \gamma^{s+1}\ln(h_0) + \sum_{i=0}^{s}\gamma^{s-i}\lambda_i + \sum_{i=0}^{s}\gamma^{s-i}\varepsilon_i$$

The relation is true for $t = s+1$, and holds therefore for any t.

Denoting $x_t = \ln(H_t)$ et $m_t = \sum_{i=0}^{t}\gamma^{t-i}\lambda_i$, and with $\lim_{t\to\infty} E\left(\sum_{i=0}^{t}\gamma^{t-i}\varepsilon_i\right) = 0$, it appears that:

$$x_t = E[\ln(h_t] + \frac{\sigma_t^2}{2} \underset{t\to\infty}{\longrightarrow} \sum_{i=0}^{t}\gamma^{t-i}\lambda_i + \frac{1}{2}\frac{\omega^2}{1-\gamma^2} = \lim_{t\to\infty} m_t + \frac{1}{2}\frac{\omega^2}{1-\gamma^2} \tag{65}$$

Denoting $P(X)$ the probability of event X, one can define $\varphi_t(x_m)$ for any real x_m as the probability:

$$\varphi_{t+1} = \mathcal{P}(\ln(h_{t+1}) < x_m) = \mathcal{P}\Big(\ln(h_{t+1}) < x_m \Big| h_0\Big).\mathcal{P}(h_0) \tag{66}$$

$$\varphi_{t+1} = \mathcal{P}\Big(\sum_{i=0}^{t}\gamma^{t-i}\varepsilon_i < x_m - \gamma^{t+1}\ln(h_0) - \sum_{i=0}^{t}\gamma^{t-i}\lambda_i\Big)\mathcal{P}(h_0)$$

$$\varphi_{t+1} = \int_{-\infty}^{\infty}\Phi\left(\frac{x_m - \gamma^{t+1}\ln(h_0) - \sum_{i=0}^{t}\gamma^{t-i}\lambda_i}{\sqrt{\sum_{i=0}^{t-1}\gamma^{2i}}}\right)dG_0(h)$$

Using a Taylor development of Φ around $\chi_t = \frac{\sqrt{1-\gamma^2}}{\sqrt{1-\gamma^{2(t-1)}}}(x_m - \sum_{i=0}^{t-1}\gamma^{t-1-i}\lambda_i)$, it leads to:

$$\varphi_t = \Phi(\chi_t) + \sum_{j=0}^{\infty} \frac{\Phi^{(j)}(\chi_t)}{j!}\gamma^{tj}\left(\frac{\sqrt{1-\gamma^2}}{\sqrt{1-\gamma^{2(t-1)}}}\right)^j \int_{-\infty}^{\infty} \ln(h_0)^j dG(h_0) \tag{67}$$

As $\int_{-\infty}^{\infty}(\ln(h_0 - E[\ln(h_0])^j dG(h_0)$ is bounded, every moments of the initial distribution are bounded for any j, by $\mathcal{M}$, the maximal moment of the initial distribution.

$$\left|\int_{-\infty}^{\infty}(\ln(h_0)^j dG(h_0)\right| < \mathcal{M}|\sum_{i=0}^{j}\left(\begin{array}{c} j \\ i \end{array}\right) E[\ln(h_0]^i| = \mathcal{M}(E[\ln(h_0] + 1)^j$$

$$\Rightarrow |\varphi_t - \Phi(\chi_t)| < \gamma^t \mathcal{M} \sum_{j=1}^{\infty} \frac{\Phi^{(j)}(\chi_t)\,(E[\ln(h_0] + 1)^j}{j!} = \gamma^t \mathcal{M}\left(\Phi\left(\chi_t + E[\ln(h_0] + 1\right) - \Phi(\chi_t)\right) \underset{t\to\infty}{\longrightarrow} 0$$

$$\Rightarrow \varphi_t \underset{t\to\infty}{\longrightarrow} \Phi\left(\sqrt{1-\gamma^2}\left(x_m - \sum_{i=0}^{t-1}\gamma^{t-1-i}\lambda_i\right)\right) = \Phi\left(\sqrt{1-\gamma^2}\,(x_m - m_t)\right) \tag{68}$$

There is a convergence for any x_m, so the distribution of human capital converges to a log-normal distribution. Thus the median: $\phi_t(x_m) = \frac{1}{2} = \Phi^{[-1]}(0)$ converges to m_t. The evolution of the asymptotic system is given by:

$$\left\{\begin{array}{l} x_{t+1} = m_t + \frac{1}{2}\frac{\omega^2}{1-\gamma^2} \\ ln(\tau_{t+1}) = \mathcal{T}(x_m) = \mathcal{T}(m_t) \end{array}\right. \tag{69}$$

Using the definitions of m_t and λ_t:

$$m_{t+1} = \lambda_{t+1} + \gamma m_t = q + \delta x_{t+1} + (\delta + \zeta)\ln\tau_{t+1} + \gamma m_t \tag{70}$$

One replaces x_{t+1} and $\ln(\tau_{t+1})$ using (69).

$$m_{t+1} = q + \delta\mathcal{T}(m_t) + \frac{\delta}{2}\sigma^2 + (\gamma + \delta)m_t \tag{71}$$

The fix points m of this process are the zero of the function $M(m)$. These equilibria are stable if and only if $M'(m) < 0$. When M has only one zero, the variable m_t converges toward this unique equilibrium, which implicates the convergence of the distribution. Let us assume that M has N zero,$\{\mu_i\}_{i=1}^N$. Due to the form of the preferences, if N is even, one of the zero is double. If μ_i is a double zero of M, $M'(\mu_i) = 0$. Let us replace μ_i by two reals, $\mu_{i'}$ and $\mu_{i''}$ such that $\mu_{i'} = \mu_{i''} = \mu_i$ and $M'(\mu_{i'}) < 0$ whereas $M'(\mu_{i'}) > 0$. There is then $K \geq 0$ such that M has $2K+1$ zero. Let us note by convention $\mu_0 = -\infty$ and $\mu_{2K+2} = \infty$. The variable m_t converges toward $\mu_{2k-1}, k \geq 1$ if asymptotically $\mu_{2k-2} < m_s < \mu_{2k}$. Eventually, the human capital distribution always converges. To determine the stationary distribution, it is necessary to simulate the process for a number a sufficiently large of periods. The position of m_a according to the threshold μ_{2i} determines the stationary values of m and then the stationary distribution.

Proof of corollary 1

Let us denote $\underline{\mu} < \mu_c < \overline{\mu}$ the three zeros of M and $\mu_t = E[\ln(h_t)]$ and $\sigma_t^2 = Var[\ln(h_t)]$. The process truly converges toward the high distribution if and only if for s sufficiently large,

$\mu_s > \mu_c$. The distribution remains log-normal during the transition. The law of evolution of the parameters is then:

$$\begin{cases} \mu_{t+1} = q + (\gamma + \delta + \zeta)\mu_t + \frac{\delta}{2}\sigma_t^2 + (\delta + \zeta)\mathcal{T}(\mu_t) \\ \sigma_t^2 = \omega^2 + \gamma^2\sigma_{t-1}^2 \end{cases} \tag{72}$$

The second relation implicates that during the transition:

$$\sigma_t^2 > \sigma^2 \Leftrightarrow \sigma_0^2 > \sigma^2 \tag{73}$$

Starting from a distribution such that $\sigma_0 > \sigma$ and $\mu_0 > \mu_c$, one can prove with a sequence that $\mu_t > \mu_c$ for any t.

$$\mu_{t+1} = q+(\gamma+\delta+\zeta)\mu_t+\frac{\delta}{2}\sigma_t^2+(\delta+\zeta)\mathcal{T}(\mu_t) > q+(\gamma+\delta+\zeta)\mu_c+\frac{\delta}{2}\sigma^2+(\delta+\zeta)\mathcal{T}(\mu_c) = \mu_c \tag{74}$$

And also for $\sigma_0 < \sigma$ and $\mu_0 < \mu_c$, the process converges toward the lowest distribution. To conclude in the other cases, let us note that $\mu_s = \sum_{i=0}^{s} \gamma^{s-i}\big(q+(\gamma+\delta+\zeta)\mu_s+\frac{\delta}{2}\sigma_s^2+(\delta+\zeta)\mathcal{T}(\mu_s)\big)$ is a continuous and strictly increasing function of μ_0 and σ_0. But for any given $\sigma_0 > \sigma$ and for any s:

$$\lim_{\mu_0 \to -\infty} \mu_s(\mu_0, \sigma_0) = -\infty \tag{75}$$

Moreover, for any $\mu_0 > \mu_c$, $\mu_s > \mu_c$. The function $\mu_s(\mu_0, \sigma_0) - \mu_c$ is a continuous and strictly increasing function of μ_0 for σ_0 given, which takes both positive and negative values. Thus there exists a single value of μ_0 denoted $\mathcal{C}(\sigma_0)$ such that $\mu_s = \mu_c$ and $\mu_s > \mu_c$ for any s if and only if $\mu_0 > \mathcal{C}(\sigma_0)$. Implicit function theorem allows to conclude that $\mathcal{C}$ is decreasing. One proves also that for $\sigma_0 < \sigma$, there exists a decreasing function $\mathcal{C}'$ that to any σ_0 links a μ_0 such that $\mu_s > \mu_c$ for any s if and only if $\mu_0 > \mathcal{C}'(\sigma_0)$. Finally the results can be summed up in the following array:

	$\mu_0 > \mu_c$	$\mu_0 < \mu_c$
$\sigma_0 > \sigma$	High equilibrium	Low Equilibrium if $\mu_0 < \mathcal{C}(\sigma_0))$
$\sigma_0 < \sigma$	High equilibrium if $\mu_0 - \mu > \mathcal{C}'(\sigma_0))$	Low Equilibrium